S102PRP/S510 MODULE 9 INTO SCIENCE

MODULE 9 NAVIGATING AROUND

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4.1

STUDY GUIDE

In this Module you investigate some of the properties of circles and spheres, by examining the size and shape of the Earth. You will be building upon and extending the skills of observing and measuring that were introduced in Module 2 and the algebra and equation manipulation that were introduced in Module 8, *Energy*. This Module will introduce the idea of a map, that is, representing a town, an area of countryside, or the whole world on a flat piece of paper. To do this the Module explains the properties of circles and spheres and how lines of longitude and latitude enable us to locate places on the Earth and calculate distances between them. You will be introduced to angles and triangles, a topic to be revisited in the next Module. In other words, we introduce you to the science of the sizes and properties of lines, surfaces and shapes in space, often called *geometry*. If you have had bad experiences of geometry and algebra in the past, we hope by now you are feeling sufficiently confident to give this approach a try!

You will need a pair of compasses (for drawing circles) and a protractor (in order to measure angles), both of which can be bought at a stationers; a couple of oranges and a knife would also be useful.

You will probably need to spend between 5-6 hours working through this Module.

1 THE EARTH: CIRCLES AND SPHERES

1.1 A MODEL OF THE EARTH

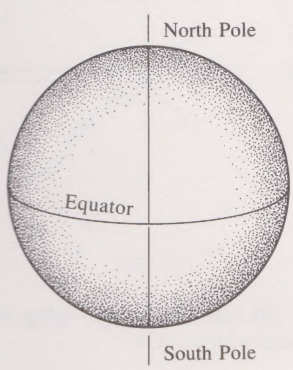


FIGURE 1 An orange used as a model of the Earth, showing the North and South Poles, the Equator and the Earth's axis.

The graphics that introduce television news programmes and photographs from satellites show us that the Earth is approximately circular in all directions, often called a **sphere**. So, we can use a smaller sphere as a representation or **model** of the Earth. A tennis ball or beach ball could be used to represent the Earth, but it is useful to have an object that can be cut up in order to examine the inside, so use an orange or a tangerine.

Figure 1 shows the orange; you will notice that it has two fixed marks on the skin, opposite each other (one is where the orange was attached to its stalk and the other is the remains of the flower). Arranging one at the 'top' of the model and one at the 'bottom', these can represent the North and South Poles of the Earth. Conventionally, on maps or diagrams such as Figure 1, North is towards the top of the figure and South towards the bottom, East is to the right and West to the left.

If you run a skewer through the North and South Poles you create a representation of the Earth's **axis**, about which it spins, completing one full turn, or revolution, each day. Of course, in reality, there is no skewer, just as there is no marked 'line' around the Earth's middle, labelled on Figure 1 as the **Equator**.

If you watch the television weather forecast you will know that above the surface of the Earth is the atmosphere, the air that we and other living things breathe; Module 7 introduced the concept of *respiration*. The atmosphere is a mixture of gases, about 80% nitrogen and about 20% oxygen (Modules 5/6) and it gets less dense at higher altitudes.

The peel and white pith of an orange can represent the Earth's atmosphere and this could be removed. If you do this very carefully you can reassemble the peel as a hollow sphere.

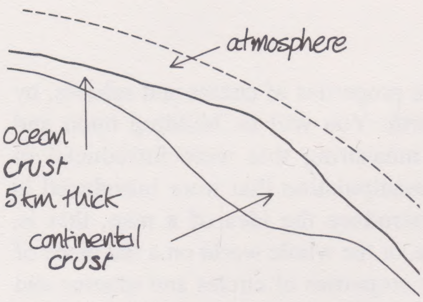


FIGURE 2 Section of the Earth, showing the atmosphere and the crust.

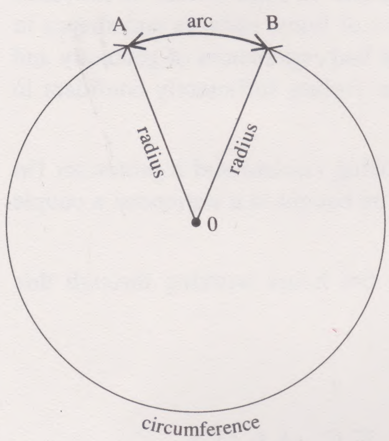


FIGURE 3 A circle, with centre, circumference, arc and radii marked.

The very thin membrane now visible around the orange represents the surface of the Earth, the rocky **crust** on which we live. It varies in thickness from about 5 km thick under the oceans to up to 60–90 km thick under the highest mountain ranges. The relationship between the atmosphere and the crust are shown in Figure 2.

1.2 CIRCLES AROUND THE EARTH

This Section continues with the use of an orange to represent a model of the Earth and considers the shapes produced when an orange is cut.

> **HOW TO DRAW A CIRCLE**
>
> If you have never drawn a circle, use your **pair of compasses** to trace around the circle in Figure 3. To do this, place the point of the compasses at the centre of the circle, and move the pencil so that it rests on the printed circle. Then, keeping the point firmly on the centre, rotate the pencil so that it traces a circle. The distance between the centre point and the pencil stays the same during the whole rotation and this is the definition of a circle: a series of points (i.e. a line called the **circumference**) at a fixed distance from a point (the centre of the circle, marked O on Figure 3) called the **radius** (plural radii). The part of the circle that joins two radii is called an **arc**; it is part of the circumference. These are shown in Figure 3.

If you cut an orange along the Equator the cut surface is a circle, shown in Figure 4a. This is called a **perspective** (or 3-dimensional) view. Figure 4b shows the view from directly above, the *plan* view.

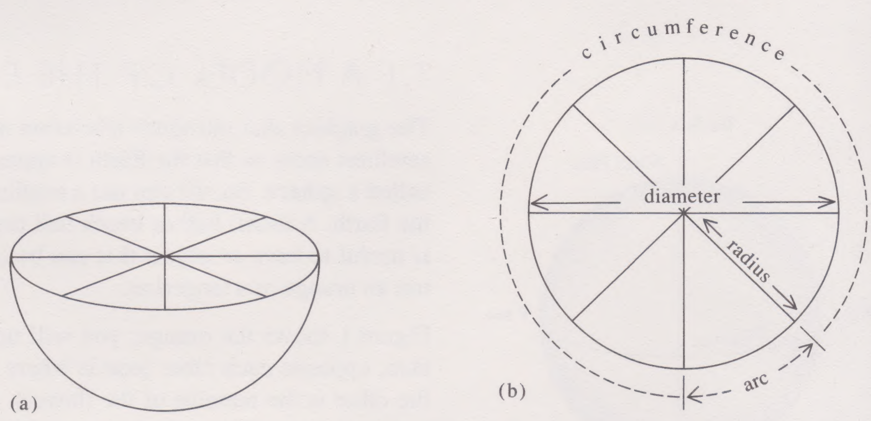


FIGURE 4 (a) Half an orange—perspective view, (b) Half an orange—plan view, showing the circumference, an arc, the radius and diameter of the circle.

It is at this point that using an orange as a model of the Earth breaks down, as the Earth doesn't have a series of spokes as shown in Figure 4b. The outside edge (the rocky sphere) is now represented by the circumference of the circle. The 'spokes' are the sides of the orange's segments; each one stretches from the centre to the circumference—they are said to spread from the centre radially, since each is a radius of the circle. A straight line from circumference to circumference, through the centre is called the **diameter**, shown in Figure 4b.

□ In terms of the radius of the Earth, how long is the diameter of the Earth?

■ The Earth's diameter is twice the size of the Earth's radius. It is always the case that the diameter of a circle is twice the radius.

We could cut the orange again, between the Equator and the pole, keeping the same distance from the Equator, as shown in Figure 5. The new cut is **parallel** to the Equator.

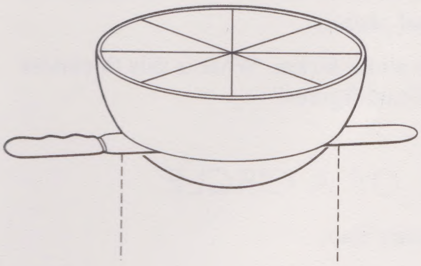


FIGURE 5 Cutting an orange parallel to the Equator.

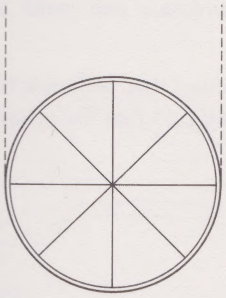


FIGURE 6 A smaller circle than the Equator.

□ What *shape* will the cut surface be?

■ A circle

□ What about the size? Is (a) the diameter of the new circle and (b) the circumference of the new circle bigger, smaller, or the same size as the circle through the Equator?

■ The diameter is smaller and the circumference is smaller; it is a smaller circle.

Circles 'around' the Earth, such as the Equator, and the smaller circle shown in Figure 6, are called **lines of latitude** and these are shown in Figure 7. Lines of latitude are used to define how far north or south a place is—compared to the reference line, the Equator.

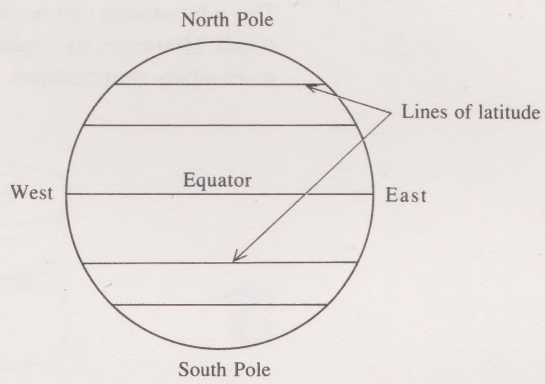


FIGURE 7 Side view of Earth showing lines of latitude.

□ Are lines of latitude all the same length?

■ No, clearly they get smaller away from the Equator, going north or south. Eventually they end as the smallest circles, points in fact, at the North and South Poles.

We could have cut the orange in half vertically, from pole to pole. If you divide the orange this way, into individual segments, each one is crescent-shaped. Starting at one pole each segment has a curved surface that extends to the other pole. The whole has the shape of the 'chocolate orange' that one can buy at sweet shops (shown in Figure 8).

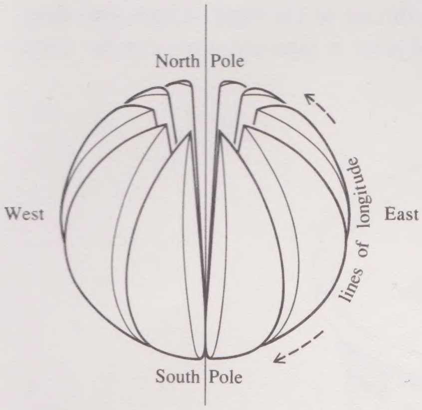


FIGURE 8 The segments of a chocolate orange.

Figure 8 shows where the edges of the segments join on the surface; these lines represent **lines of longitude**, each one joining the north and south poles.

□ Are lines of longitude all the same length, or are they of different lengths?

■ They are all the same length.

As with many frames of reference, there has to be a baseline, from which others are measured. In the case of the Earth, the reference line of longitude passes north-south through the Greenwich Observatory, on the Thames just East of London. This line is called the **Greenwich meridian.** Other lines of longitude are measured East or West of this line.

The lines of latitude and longitude mean that we can define where places are on the surface of the Earth, a bit like using graph paper. They have been used for centuries by navigators on ships to plot where they are on the Earth's surface, or a course to the next port.

SAQ 1 It was mentioned in Section 1.1 that the thickness of the Earth's crust varies; it is about 5 km thick under the oceans and, on average, 35 km thick under the continents. The Earth's radius is 6 370 km.

(a) What is the thickness of the crust beneath the oceans, as a percentage of the Earth's radius (to 3 decimal places)?

(b) What is the average thickness of the crust beneath the continents, as a percentage of the Earth's radius (to 3 decimal places)?

SAQ 2 The thickness of the atmosphere is about 80 km. What is this thickness as a ratio to the Earth's radius, to two significant figures?

1.3 THE CIRCUMFERENCE OF A CIRCLE

In this Section we shall establish by experiment that:

(i) there is a fixed relationship between the radius and the circumference of a circle and

(ii) this relationship is important for the mathematics and sums involving circles.

The relationship can be derived, but it is not important that you understand it in detail. However, by reading the derivation you may find that the relationship is more easily remembered.

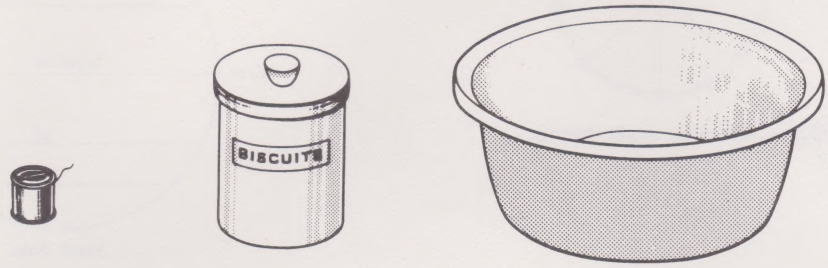


FIGURE 9 Several objects, each with a circular base.

In order to do this, you need to collect a number of circular objects, for example some of those shown in Figure 9. For each of the objects you have collected, measure the *shortest* length *round* the base and the *longest* length *across* the base. You will probably find centimetres the most convenient units to use for your own measurements. For the smallest objects (e.g. a cotton reel) you will probably have to cut a piece of string or thread to the right length and then measure this against a ruler; on larger objects a tape-measure can be used directly (Figure 10).

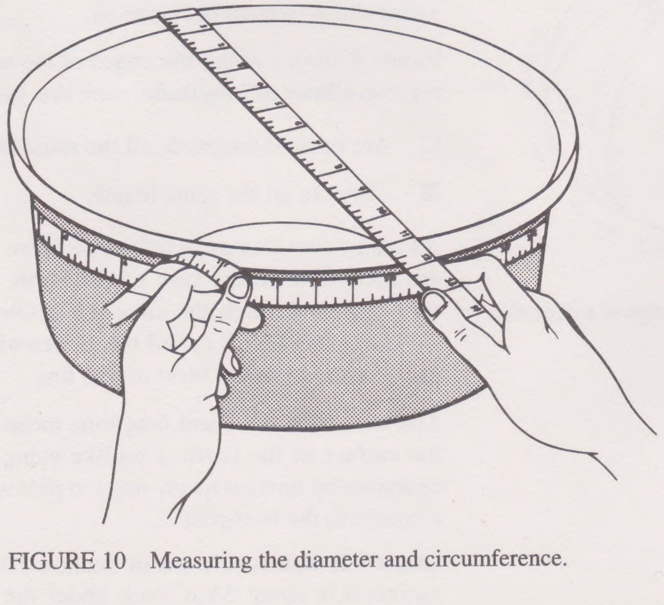


FIGURE 10 Measuring the diameter and circumference.

The first length to be measured will be the circumference of the circular base; we represent this by the letter C. The second length is the diameter of the base and we represent this by the letter d. Table 1 shows some sample measurements

of C and d for a few objects. The object of the exercise is to find out the ratio C/d for each set of values (as shown in the extreme right-hand column of Table 1): providing C and d are in the same units, their ratio will be without units, as when C (cm) is divided by d (cm) the answer is a number without any unit. (Recall from Module 2 that ratios do not have units):

$$\frac{C\,\text{cm}}{d\,\text{cm}} = \frac{C}{d}$$

TABLE 1

Object	C/cm	d/cm	Ratio $\frac{C}{d}$	Rounded to appropriate sig figs
mug	25.8	8.2	3.15	3.2 (two sig figs)
glue-pot	8.0	2.5	3.20	3.2 (two sig figs)
dust-bin	100.0	33.0	3.03	3.03 (three sig figs)

Enter your own measurements in the bottom half of Table 1 and calculate the corresponding values of the ratio C/d.

□ What do you notice about the ratio C/d?

■ You should find that in all cases the ratio is approximately equal to 3

What you have worked out is an important mathematical ratio. The ratio of the circumference to the diameter is always the same, for all circles. When a relationship, such as this ratio, is found to be the same on all occasions it is called a **constant** and in this case it is denoted by the Greek letter π (**pi,** pronounced 'pie'). The value of π is not an exact number: it has an unending sequence of numbers after the decimal point. Mathematicians have used computers to calculate π to thousands of decimal places. Your calculator will have a key marked π which will display the value to many decimal places. To six significant figures it is 3.141 59 (see Box 1). However, you may never need to be as precise as this: for most practical purposes it is sufficiently accurate to use $\pi = 3.14$ (to three significant figures). It is also often useful to have π in fractional form, and the approximation $\pi = 22/7$ is worth remembering. You do *not* have to remember the value in decimals. In this Module we use both 22/7 and the π button on the calculator. Although the latter is straightforward we use 22/7 to give you practice at using fractions. Note that by assuming $\pi = 22/7$ to be *exact*, its value when expressed in this way does not affect the number of significant figures in any calculation.

BOX 1

Press π 3.141 592 654 should appear

So, the circumference, C, and the diameter, d, of a circle are related by the equation:

$$\frac{C}{d} = \pi$$

This equation can be used to find the length of the circumference, if you know the diameter of a circle.

□ Rearrange this equation so that C is the subject (i.e. $C = \ldots$)

■ The equation may be rearranged by multiplying both sides by d to give:

$$C = \pi d \qquad (1)$$

But by definition the diameter is twice the radius, so

$$d = 2r \qquad (2)$$

□ Write down the equation for C, in terms of r and the constant π

■ Replacing d in equation (1) by $2r$, the equation is

$$C = \pi(2r)$$

i.e. $C = 2\pi r$

The circumference of a circle = $2\pi r$

(You say this equation as 'circumference equals two pie are'). Note that it is usual in a formula like $2\pi r$ to put numbers first (the 2), followed by any constants (π) and finally algebraic symbols, called variables (in this case r standing for the values that the radius can have).

Using these equations, you should now be able to calculate the circumference of a circle given either its diameter or its radius, and vice versa.

□ What is the value of the Earth's circumference, if π is 22/7 and the radius is 6 370 km? Answer to four significant figures.

■ $C = 2 \times 22/7 \times 6\,370\,\text{km}$

$= 40\,040\,\text{km}$

The circumference of the Earth is about 40 040 km

You should note that the units of circumference, radius and diameter are all those of length: mm, cm, m, or even km.

□ Make the radius, the subject of the equation $C = 2\pi r$

■ $r = C/2\pi$

GUIDED EXERCISE

The aim of this exercise is to show you how to divide by a fraction. Suppose you wanted to work out the radius of a circle of circumference 108 m to three significant figures.

$$C = 2\pi r$$

$$r = \frac{C}{2\pi}$$

$$= \frac{108\,\text{m}}{2\pi}$$

$$= \frac{108\,\text{m}}{2 \times \frac{22}{7}}$$

Now, you have not met a situation where you have to *divide* by a fraction. Try an everyday example: you have two oranges, each to be divided into four pieces, each 1/4 of the whole orange.

□ How many pieces in two whole oranges?

■ Eight

Expressed mathematically this is:

$$2 \div \frac{1}{4} = 8$$

Can you see how this was done? The fraction you divide by (the denominator) is inverted (turned the other way up) and the divide is changed to *multiply*:

$$2 \div \frac{1}{4} = 2 \times \frac{4}{1}$$

Try another one.

□ What is five-eighths divided by a quarter?

■ Again, the fraction that you divide by—called the divisor—is inverted and changed to multiply:

$$\frac{5}{8} \div \frac{1}{4} = \frac{5}{8} \times \frac{4}{1}$$

Now we can 'cancel' (divide) top and bottom by 4, so this reduces to

$$\frac{5 \times 1}{2 \times 1} = \frac{5}{2} = 2.5$$

This is an important rule that you should try to remember:—

> To divide by a fraction, invert the divisor and multiply

Now to return to our Guided Exercise.

$$= \frac{108\text{ m}}{2 \times \frac{22}{7}}$$

$$= \frac{108}{2} \times \frac{7}{22}\text{m}$$

= 17.181 818 m (17.2 m to three significant figures).

If you did not get this answer, check your method with that given in Box 2.

BOX 2
Press 108
press ×
press 7
press ÷
press 2
press ÷
press 22
press = 17.181 818 should appear

For SAQs 3 and 4, use $\pi = 22/7$

SAQ 3 A circular cake tin has a diameter of 20.0 cm. To the nearest cm what length of paper is required to line the vertical side, assuming no overlap?

SAQ 4 Britain is some way north of the Equator, but south of the North Pole, as shown in Figure 11. A person travels east from London, along a constant line of latitude and eventually arrives back at their starting point. The distance travelled was 25 737 km (to 5 significant figures). What is the length of the radius of the circle along which they travelled?

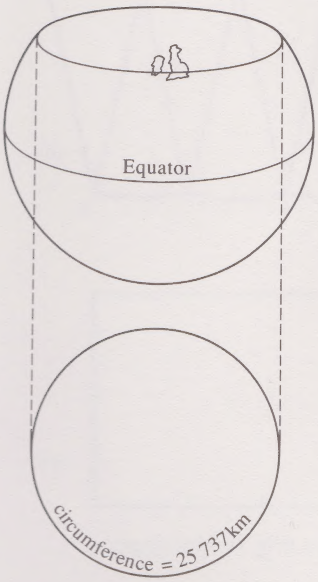


FIGURE 11 A line of latitude with circumference 25 737 km.

For SAQ 5 use π on your calculator.

SAQ 5 The Earth has a radius of 6 370 km and spins once on its axis every 24 hours. Find the speed at which a place on the Equator moves. (Hint; how far does the place travel in exactly 24 hours? So what is its speed in km per hour? Give your answer in scientific notation to three significant figures.)

1.4 THE AREA OF A CIRCLE

You have calculated the area of rectangles in previous Modules, but calculating the area of a circle is less simple. In this Section we derive an expression (or *formula*) for the area of a circle, by a 'paper experiment'. You do not have to remember how the formula is derived but you will find it easier to remember the result (the formula) if you follow how it was obtained.

Look at the circle of radius r shown in Figure 12. It is similar to Figure 4b, that of an orange, showing the internal segments.

The circle in Figure 12 has been divided up into eight segments or *sectors*. If you were to cut out these sectors, you could rearrange them into the shape of Figure 13a.

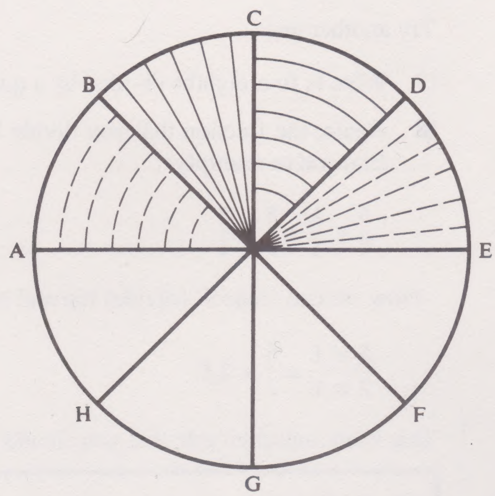


FIGURE 12 Circle marked with sectors.

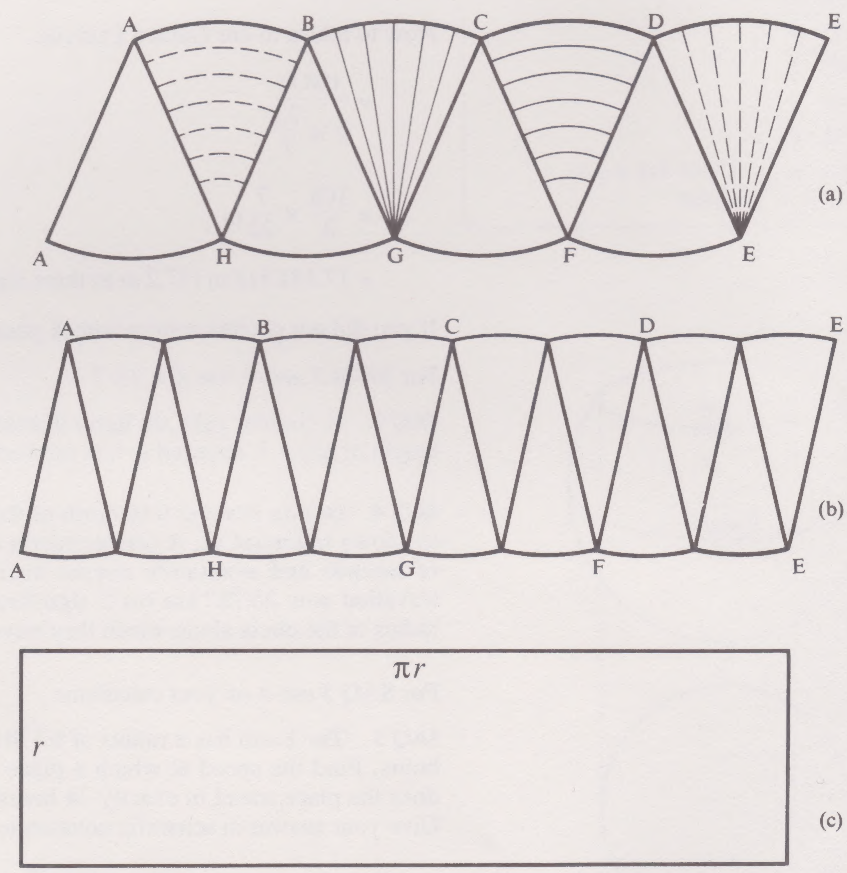


FIGURE 13 (a) Sectors in a strip, (b) Smaller sectors in a strip, (c) Rectangle.

The length AE (measured along the curved edge, the four arcs) is half the circumference of the circle, (i.e. πr) and the length AA or EE is the radius r. If you were now to cut each sector in half, you could produce the shape of Figure 13b. This is similar to (a), but the 'scalloped' appearance of the line AE is less curved, and AA and EE are more nearly perpendicular to AE. If you were to repeat the halving and rearranging process again and again, AE would get nearer and nearer to a straight line and more and more nearly perpendicular to AA and EE. If you persevered long enough, you could eventually reassemble the infinitely narrow sectors into the rectangle of Figure 13c. This rectangle has length πr and breadth r.

□ What is the area of the rectangle show in Figure 13c? The formula for the area of a rectangle was given in Module 3. It is area = length × breadth.

■ The area of the rectangle in Figure 13c is $(\pi r) \times r = \pi r^2$.

This rectangle was obtained by cutting up the circle, so the area of the circle must be equal to that of the rectangle. Thus:

$$\boxed{\text{area of a circle} = \pi r^2}$$

You should be able to use this formula to calculate the area of a circle given its radius (or diameter) and vice versa.

□ Let A stand for the area of a circle. Rearrange the equation $A = \pi r^2$ so that r^2 is the subject of the equation

■ The necessary rearrangement may be achieved by dividing both sides by π to obtain:

$$r^2 = \frac{A}{\pi}$$

As you see r^2 in the formula for area, the units involved are two-dimensional mm^2, cm^2, m^2, km^2. You first met this in Module 3, when working out the area of a room, or a wall. You will be familiar, in everyday life, with buying some things in **linear** measure, e.g. metres of electric cable; whereas some things are bought and costed in area measure e.g. square metres of carpet.

Example 1 What is the area of a circle of radius 7 cm? (Take $\pi = 22/7$)

$$\text{Area} = \pi r^2 = \left(\frac{22}{7}\right) \times (7 \times 7)\,\text{cm}^2$$

As the number 7 appears in the top and bottom of the calculation it can be 'cancelled' and the calculation reduces to:

$$\text{Area} = 22 \times 7\,\text{cm}^2 = 154\,\text{cm}^2$$

Example 2 A circle has an area of 15 400 mm^2. What is its radius? (Take $\pi = 22/7$)

$$r^2 = \frac{A}{\pi}$$

Putting in the values for A and π (often called **substituting**)

$$r^2 = \frac{15\,400}{1} \div \left(\frac{22}{7}\right)\text{mm}^2$$

$$r^2 = 15\,400 \times \frac{7}{22}\,\text{mm}^2$$

$$r^2 = 4\,900\,\text{mm}^2$$

To get r, we need to take the square root of each side of the equation: (Remember that square root was introduced in Module 3)

$$\text{Therefore } r = \sqrt{4\,900\,\text{mm}^2} = 70\,\text{mm (see Box 3)}$$

$$\text{so } r = 70\,\text{mm}$$

BOX 3
Press 4 900
press √ 70 should appear

SAQ 6 A lighthouse surrounded by sea can be seen up to a distance of 21.0 km. What is the area of the circle in which it can be seen?

(a) Take $\pi = 22/7$ and give your answer to three significant figures.

(b) Use the π button on your calculator and give your answer to three significant figures.

SAQ 7 What is the diameter of a circle of area 401 cm^2?

(a) Use $\pi = 22/7$ and give your answer to the appropriate number of significant figures.

(b) Use the π button on your calculator and give your answer to an appropriate number of significant figures.

SAQ 8 A farmer has 400 m of fencing. Will his sheep have a larger area of grass on which to feed and by how much, if he uses this to enclose a square pen rather than a circular one? (Specify the difference in m^2 to three significant figures.) Use the π button.

1.5 THE SURFACE AREA OF A SPHERE, THE EARTH

You have learnt how to calculate the area of a circle. This Section shows you how to calculate the surface area of a sphere.

A sphere, such as an orange, or the Earth, is a three-dimensional circle. We shall not derive the formula for the surface area of a sphere, but just give it to you to use:

$$\boxed{\text{Surface area of sphere} = 4\pi r^2}$$

You can see that it is similar to the formula for the area of a circle, including the units, mm^2, cm^2, m^2, km^2 as we are still dealing with an area.

□ The diameter of the Earth is 12 740 km, what is its surface area? Write down an expression but don't do the final calculation.

■ The area is given by the formula $4\pi r^2$ The radius is 6 370 km, so the area

$$A = 4 \times \pi \times (6\,370)^2\,(\text{km})^2 \qquad (3)$$

Now, if you punch 6 370 into your calculator and multiply it by itself or use the 'squared' button, you are going to get a large number, especially when you then multiply by 4π. So, you should first change 6 370 into scientific notation.

□ Express 6 370 in scientific notation

■ 6.37×10^3

So equation (3) becomes

$$A = 4 \times \pi \times (6.37)^2 \times (10^3)^2\,\text{km}^2$$

□ What is a simpler form of $(10^3)^2$? (i.e. remove the brackets)

■ 10^6. This is because $(10^3)^2 = 10^3 \times 10^3 = 10^6$

Now, check the calculation in Box 4 on your calculator

BOX 4

Press 4	
press ×	
press π (or 'exp' button)	(3.141 592 7 should appear)
press ×	(12.566 371 should appear)
press 6.37	
press x^Y	
press 2	
press =	(509.904 36 should appear)

So, Area = $509.904\,36 \times 10^6\,\text{km}^2$

$= 510 \times 10^6\,\text{km}^2$ (to 3 sig figs)

Or, in words, five hundred and ten million square kilometres.

□ Express the answer in scientific notation

■ $5.10 \times 10^8\,\text{km}^2$ (to three sig figs)

You can now calculate the area of, for example, the land masses and the oceans on the Earth's surface.

□ If 70% of the surface area of the Earth is covered by the oceans, what is their area in km^2?

■ Area = total surface area × 70%

$= \frac{70}{100} \times 5.10 \times 10^8\,\text{km}^2$

dividing by 100 reduces 10^8 to 10^6 so area

$= 70 \times 5.10 \times 10^6\,\text{km}^2$

$= 357 \times 10^6\,\text{km}^2$ (to 3 sig figs)

$= 3.57 \times 10^8\,\text{km}^2$ (to 3 sig figs in scientific notation).

1.6 THE VOLUME OF A SPHERE, THE EARTH

The Section above showed you how to calculate the surface area of a sphere. This Section looks at how to calculate the volume.

The volume of a sphere is given by the formula:

Volume of sphere = $4\pi r^3/3$

This time, because we are dealing with volume, the dimensions are cubic, i.e mm^3, cm^3, m^3, km^3. These units were introduced in Module 3 and are the usual measure for solids; you may be more familiar with the volume units for liquids, litres; 1 litre = 1 000 cm^3.

□ The diameter of an orange is 10.0 cm, what is its volume? (Use the π button and give your answer to 3 significant figures).

■ The volume is given by the formula $4\pi r^3/3$. The radius is 5.00 cm,

so the volume $= \frac{4}{3} \times \pi \times (5)^3\,\text{cm}^3$

$= 523.60\,\text{cm}^3 = 524\,\text{cm}^3$ (to 3 sig figs)

The keys to press on your calculator are shown in Box 5.

BOX 5

Press	4	
press	×	
press	π	
press	×	
press	5	
press	x^Y	
press	3	
press	=	(1 570.7963 should appear, the numerator of the calculation)
press	÷	
press	3	
press	=	(523.598 78 should appear)

□ The radius of the Earth is 6 370 km and π = 22/7. What is the volume of the Earth? Write down the expression, but follow the calculation below.

■ The volume, V, is given by the formula $4\pi r^3/3$

$V = \frac{4}{3} \times \pi \times (6\,370\,\text{km})^3$

$= \frac{4}{3} \times \pi \times (6\,370)^3\,\text{km}^3$

Then, recall the way we approached the calculation of the area, by first calculating r^2, or in this case, $r^3 = (6.37 \times 10^3)^3$. This is the same as $(6.37)^3 \times (10^3)^3$.

□ Write a simpler expression for $(10^3)^3$ i.e. without the brackets.

■ 10^9, because $(10^3)^3 = (10^3 \times 10^3 \times 10^3)$

To calculate $(6.37)^3$ you need to use your calculator, as shown previously in Box 5.

BOX 6

Press	4	
press	×	
press	π	
press	×	(12.566 371 should appear)
press	6.37	
press	x^Y	
press	3	
press	=	(3 248.090 8, the numerator of the calculation will appear)
press	÷	
press	3	
press	=	(1 082.696 9 should appear)

The volume is:

$$V = \frac{4\pi}{3} \times (6.37)^3 \times 10^9\,\text{km}^3$$

$$= 1\,082.70 \times 10^9\,\text{km}^3 \text{ or } 1.08 \times 10^{12}\,\text{km}^3 \text{ (to 3 sig figs)}$$

The calculator keys you need to use are shown in Box 6.

Now that you have learned how to calculate the area and volume of spheres, such as the Earth, we will tell you more about its internal structure in the next Section.

1.7 THE RUSSIAN DOLL MODEL

Have you seen a picture of a carved Russian doll with a join around the middle, like the one in Figure 14a? When you pull the two sections apart there is an identical, but smaller doll inside. The Earth is a bit like a spherical Russian doll, with the Equator as the join: you already know about the atmosphere and the crust, but what about *inside* the Earth? The rocky crust is only a small fraction of the whole thickness, so what lies beneath?

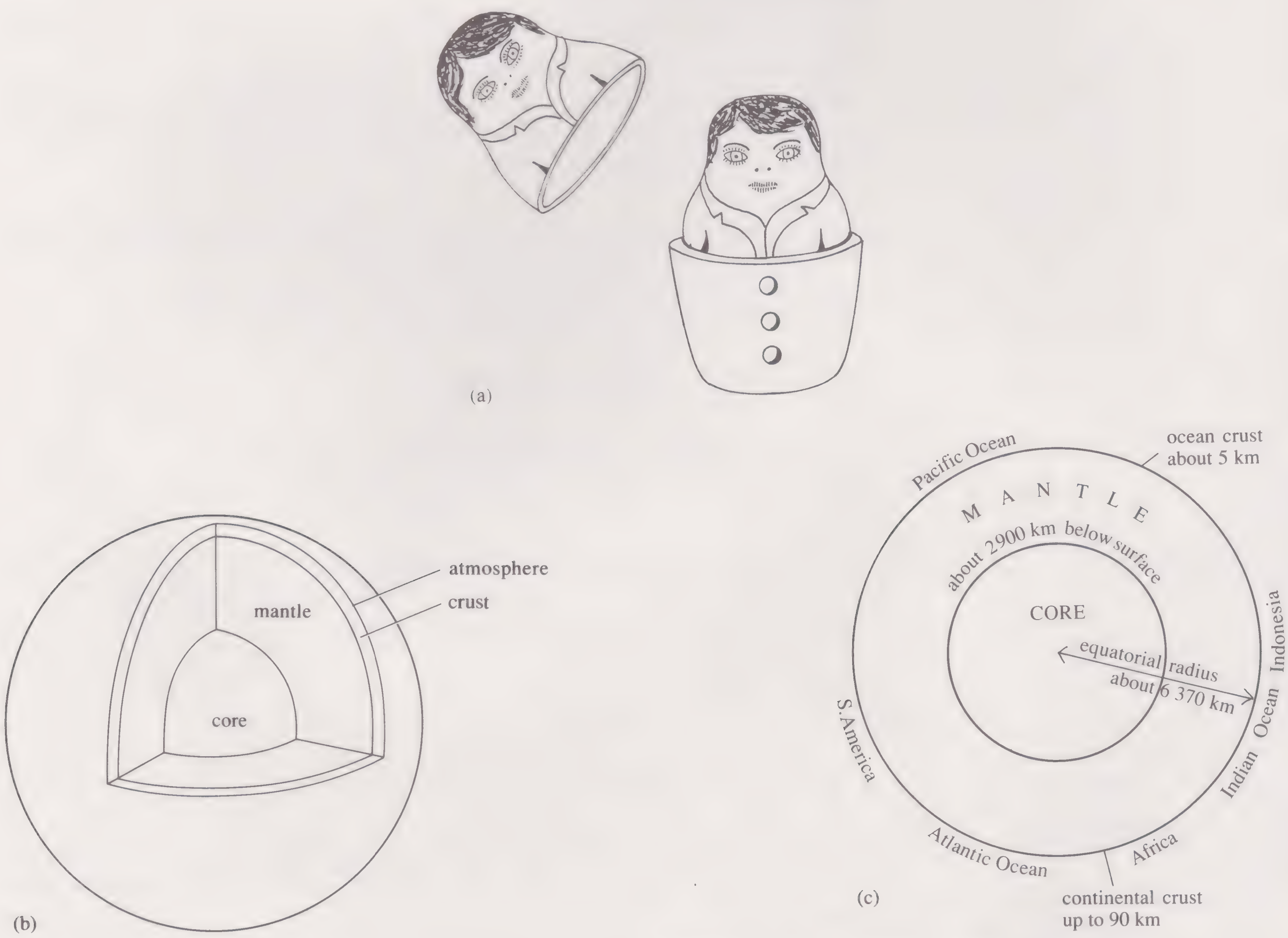


FIGURE 14 (a) A Russian doll, (b) Internal structure of the Earth, (c) Internal dimensions of the Earth.

The structure of the interior of the Earth is indicated in Figure 14b; it is a series of spheres, one inside the other, like a giant aniseed ball.

Immediately beneath the crust is the **mantle**, a thick layer of solid rock that has a higher density than rocks found at the surface of the Earth. The mantle is the greatest part of the volume of the Earth. As the temperature increases the deeper you go into the Earth, these rocks are also hotter than those found at the Earth's surface.

□ What evidence can you think of that supports the idea that it is hotter deeper in the Earth?

■ Volcanoes indicate that there are at least some areas beneath the surface where temperatures are higher. You may also know that the temperatures in deep mines are higher than surface temperatures.

At the centre of the Earth is the **core** which scientists believe has two parts, an outer part that is liquid and an inner part that is solid. Both are believed to contain a high proportion of iron. It is necessary to say 'believed' because our direct evidence of the interior of the Earth only stretches down to about 12 km, a mere scratch in comparison to the Earth's radius of 6 370 km. All our knowledge comes from *indirect* evidence, particularly the study of earthquakes; as you will learn in later studies.

SAQ 9 What percentage of the volume of the whole Earth is:

(a) the core,

(b) the mantle? (Hint: the volume of the mantle = vol of Earth − volume of core),

Use the distances marked on Figure 14c and assume the volume of the Earth is:

$$1.08 \times 10^{12}\,\text{km}^3\text{, or } \frac{4}{3}\pi\,(258.47 \times 10^9)\,\text{km}^3$$

Note: the crust is only about 1% of the Earth's volume, so we are ignoring it for these calculations.

2 THE EARTH: ANGLES

You now know about the size and shape of the Earth, but before we can start making maps or charting a course on the surface you need to know about another concept: that of *direction* and to measure this we use the concept of **angle**. In this Section we shall consider units of angle, and will use a protractor actually to measure them.

2.1 DEFINITION OF ANGLE

What do you understand by the term 'angle'? The *Oxford English Dictionary* defines it as 'space between two meeting lines or planes; inclination of two lines to each other; corner'. In fact, the English word is derived from the Latin *angulus,* meaning corner. Our definition may be summed up by saying that an angle is a measurement of an amount of rotation.

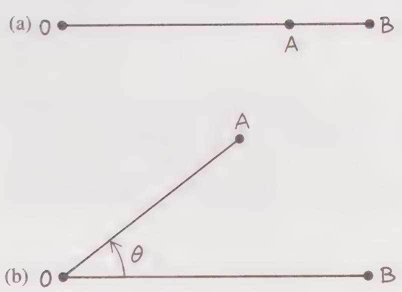
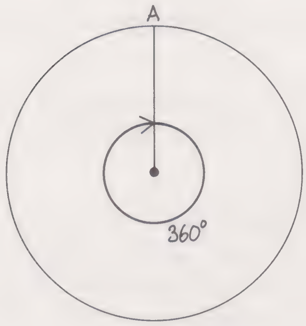


FIGURE 15 (a) Two lines, one on top of the other, (b) Two lines making an angle.

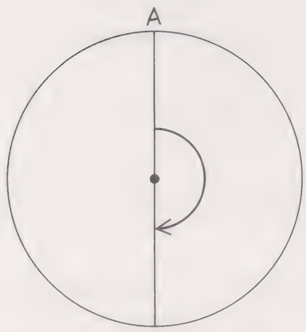
Figure 15a shows two lines, OA and OB, lying one over the other, like the hands on a clock. If the line OA is then rotated in an anti-clockwise direction using the point O as a pivot, it is said to turn through an angle (Figure 15b). Angles are often labelled with Greek letters: the one formed as a consequence of the rotation of OA is labelled θ (theta—pronounced with th as in *th*ink, et as *eat* and ending with short a; the accent is on the first syllable), in Figure 15b.

There are a number of different shorthand notations that may be used to refer to an angle. The angle between the lines OA and OB can be denoted by the symbol ∠AOB. Notice that the point which is common to both lines is in the middle. This point is called the **vertex**. If there is only one angle with vertex O, this angle is sometimes simply denoted as ∠O.

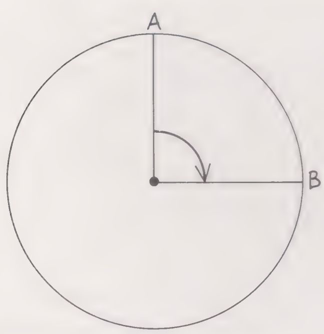
Some of the most commonly used letters used to represent angles are: α (alpha), β (beta), γ (gamma), θ (theta) and ϕ (phi, pronounced 'fie').



(a)



(b)



(c)

FIGURE 16 Angles in a complete revolution (a) 360°, (b) 180°, (c) 90°.

2.2 UNITS AND MEASUREMENT OF ANGLE

One of the most common units which can be used to measure angles are 'degrees'. One degree is defined as being 1/360 of a complete revolution, that is, if the radius of a circle describes a complete rotation, it has turned through 360 degrees, written in shorthand as 360°. This is shown in Figure 16a.

□ How many degrees are there in half a revolution?

■ 180, shown in Figure 16b.

One quarter revolution 360/4 is 90° or a right angle (Figure 16c), first introduced in Module 3 as the angle between vertical walls and horizontal floors. Imagine that you are standing in Central Milton Keynes, facing North, as shown in Figure 17a. You then turn through 90° to your right (i.e. clockwise) to face John Lewis.

(a)

(b)

FIGURE 17 Illustrations of bearings (a) Person facing North, towards C&A, (b) Person turns East to face John Lewis.

□ In which direction are you now facing?

■ East, as shown in Figure 17b.

Another way of saying this is that East is at a bearing of 090° from you. Bearings always have three figures, between 000 and 360 and they are measured clockwise from North.

It is by using bearings and distances that aircraft and ships are navigated. Closer to home you may know of the sport called 'orienteering' where people navigate around a course, usually over rough countryside, using a **compass** and a **map**. To do this they use bearings and distances; the compass enables them to find North and to plot a course on a chosen bearing.

Suppose that you were planning an orienteering course on a map. How can you find out the exact sizes of the angles of, for example, the pathway at C, relative to the fence, shown in Figure 18. To measure an angle you use an instrument called a **protractor** shown in Figure 19.

The protractor shown here is graduated to measure angles from 0° to 180°; there are also circular protractors which will measure angles up to 360° directly.

Use your protractor to measure the angle shown in Figure 20a. What is the angle?

Figure 20b shows the protractor in place ready for the measurement. The 'base line' of the protractor is superimposed on one of the lines forming the angle and the point O is placed on the vertex. The size of the angle (35°) is read off the *inside* scale, shown in Figure 20b; the outside scale would be used for measuring angles from the left-hand side of the base line in a clockwise direction.

Exercise: Use a protractor to measure the angles in Figure 21 (if the lines forming the angle are not long enough to reach the scale on your protractor, extend them).

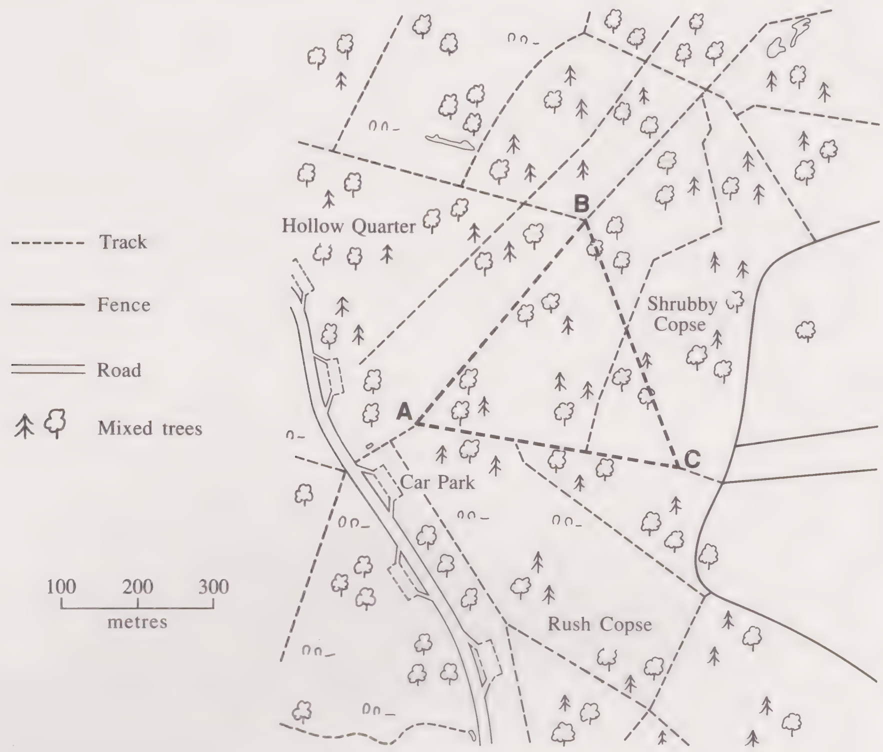


FIGURE 18 Map of a wood, part of an orienteering course.

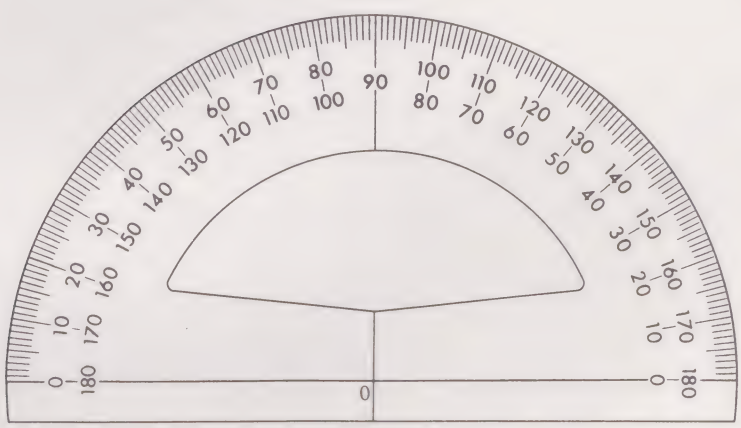


FIGURE 19 A protractor.

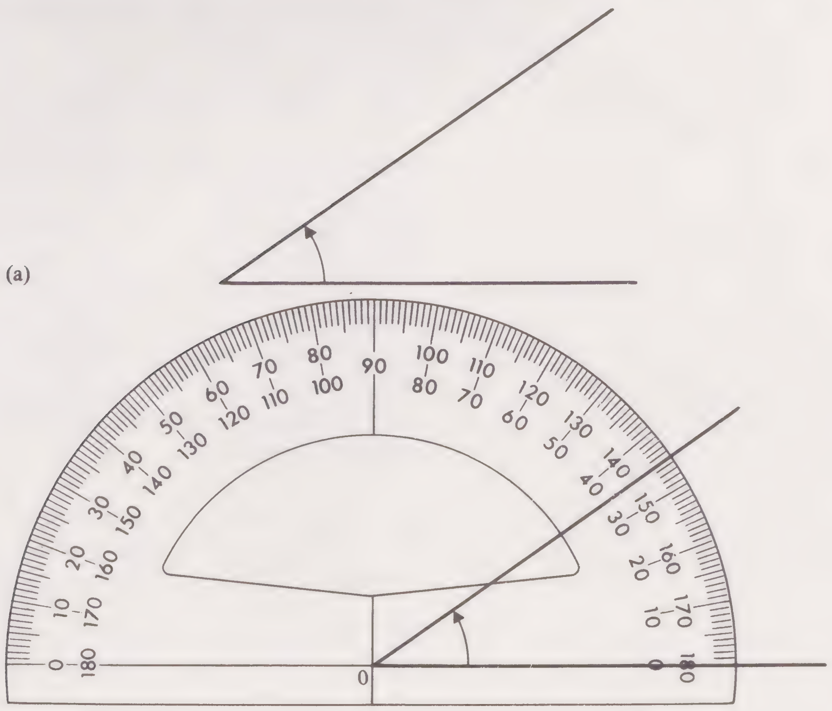


FIGURE 20 (a) Angle to be measured. (b) Angle with protractor in place.

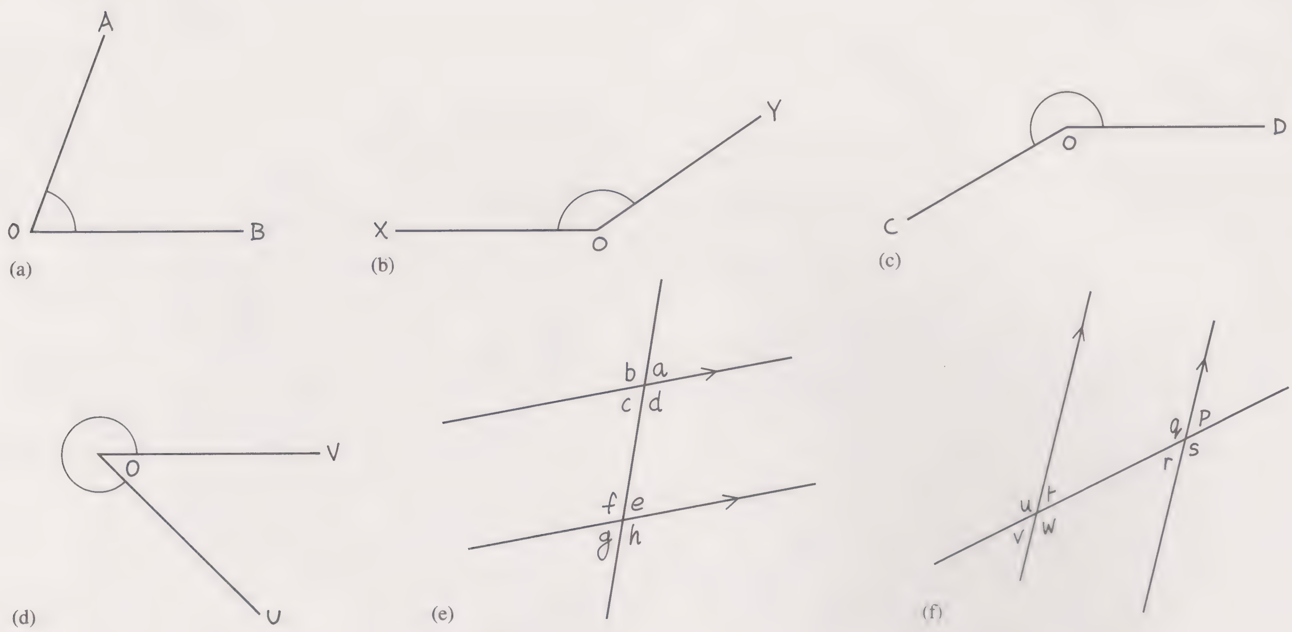


FIGURE 21 Angles to be measured with a protractor.

In Figure 21, e and f the two lines marked with arrows are parallel lines, i.e. they are always the same distance apart. Because the lines are parallel, there are pairs of angles in the diagram that are the same size. You should have found that angles *a* and *c* for example, are equal; these are called *vertically opposite* angles. Another pair of equal angles is *a* and *e*, and another pair is *c* and *e*.

SAQ 10 In Figure 21f, identify pairs of equal angles.

SAQ 11 The hands of a clock read 4 o'clock.

(a) Draw a small diagram to show the angle between the hands

(b) What is the angle between the hands?

(c) If the number 12 on the dial represents North with respect to the centre of the clock, what is the bearing of the number 4 on the dial?

With the knowledge of angles we are now able to label the lines of latitude and longitude on a map of the Earth. The next Section explains how this is done.

2.3 LABELLING LINES OF LONGITUDE

Imagine the plan view of the Earth looking down over the North Pole. It would look just like Figure 4b or 22a. The base line, the Greenwich meridian, can be labelled as 0°. The lines of longitude are measured East and West of this line, as shown in Figures 22a and 22b, where they have been marked every 30°. When you look at a map or diagram like this it is the convention to have East on the right and West on the left. Figure 22b shows that degrees of longitude refer to the angle that is made by two Earth radii at the centre of the Earth, just like the orange segments that we examined earlier. Figure 22c shows that the British Isles lie between about 1.5° East (the coast of East Anglia) and about 10° West (the west of Ireland).

□ If lines of longitude are measured East and West from zero (the Greenwich meridian), for example, 10° W on the 'left' is mirrored by 10° E on the 'right', what is the largest value for a line of longitude?

■ 180°, because you would have reached the opposite side of the circle. (You can confirm this by using your protractor on Figure 22a.

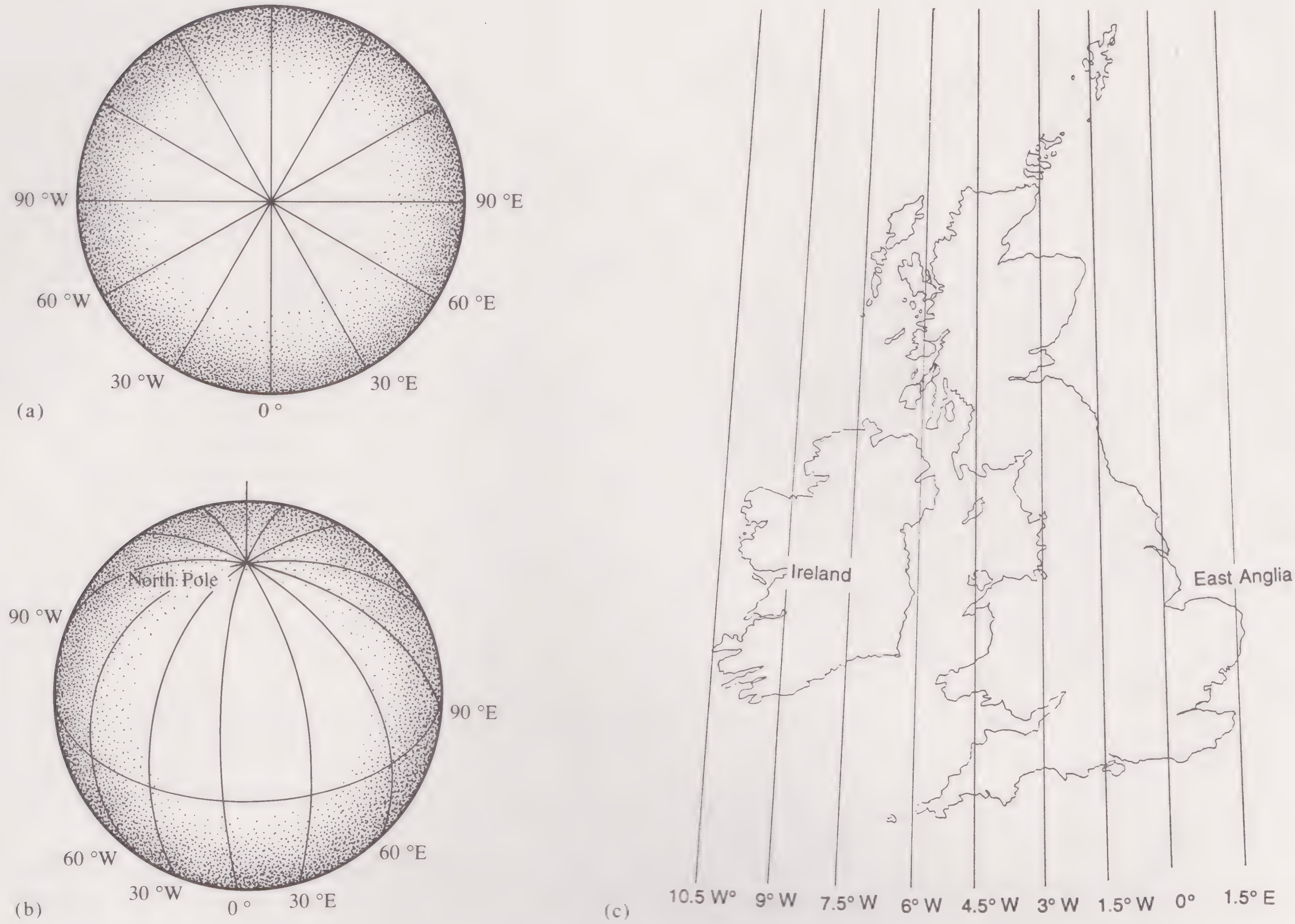


FIGURE 22 (a) Plan of the Earth cut through the Equator, lines of longitude marked every 30°, East and West of Greenwich, (b) Perspective view, lines marked at 30 degree intervals, (c) Position of Britain, lines of longitude marked.

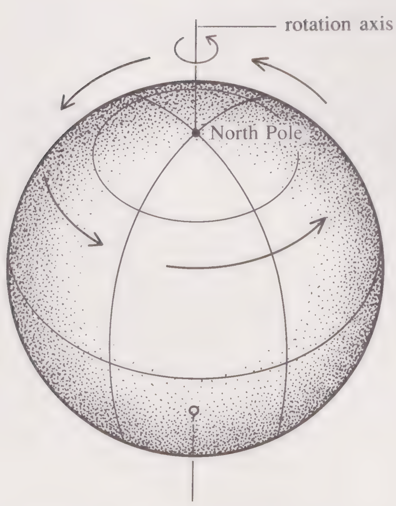


FIGURE 23 The Earth spinning anti-clockwise about the North Pole.

The line 180° is sometimes called the **International Date Line** and it runs north-south from the North Pole, west of Alaska to the South Pole in Antarctica, mostly through the Pacific Ocean. Flying across this line from East to West (e.g. Hawaii to Japan) you go to the next day. Travelling in the opposite direction, you go to the previous day, so you could take off from Japan on Saturday and land in Hawaii on the day before, Friday!

The Earth rotates on its axis (Figure 23) once each day. Looking from above the North Pole, it appears to spin in an anti-clockwise direction.

□ How many degrees of longitude does it move through every hour, if it travels through 360° in 24 hours?

■ In one hour it travels through (360/24)° = 15°

This means that roughly every 15° of longitude 'time' is an hour different and this accounts for the several 'time zones' of large countries, such as the USA and Canada. This is also why, if you travel East or West across several lines of longitude you have to change your watch.

SAQ 12 The Super Bowl (US football equivalent of the Cup Final) is sometimes held at the Super Dome arena in New Orleans, USA, (longitude, 90° W). This city is marked as NO on Figure 24.

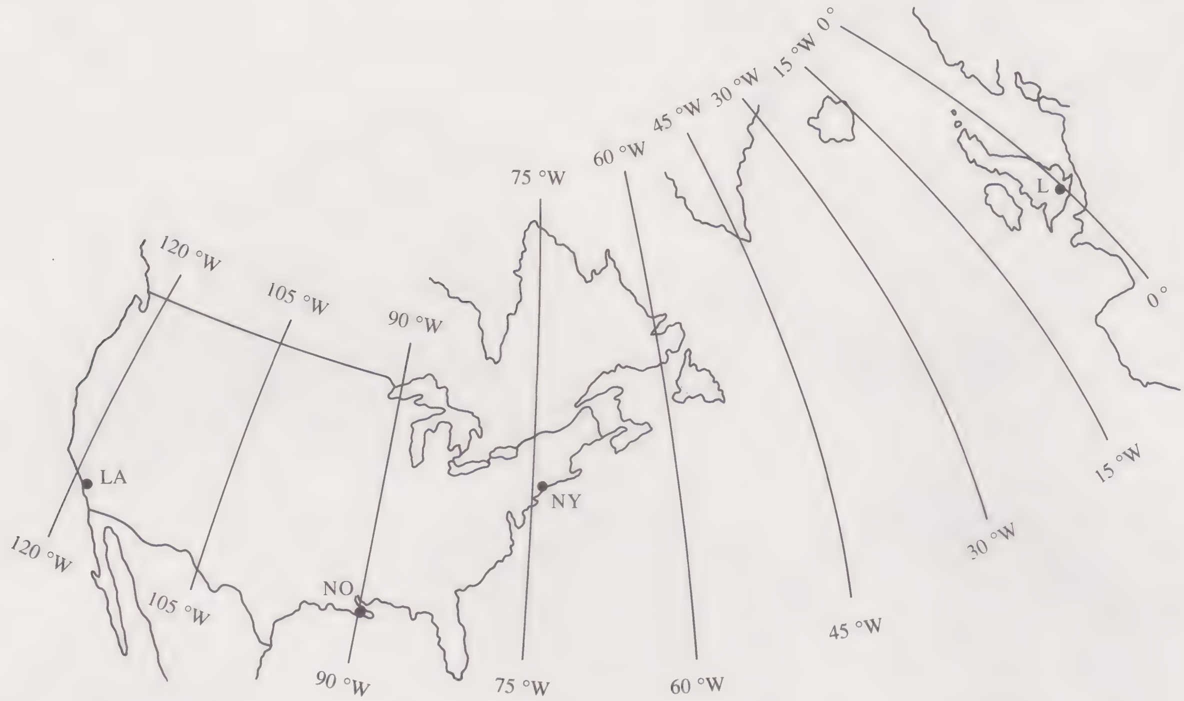


FIGURE 24 Relative positions of London, New York, New Orleans and Los Angeles.

(a) How many hours time difference is there between New Orleans and London (0°)?

Because of its position midway across the US time zones, TV transmission with the 'run-up to the Super Bowl' starts on a Sunday at 4.00pm (16.00), when the time in New York (about 74° West, NY on Figure 24) is 5.00pm and in Los Angeles (118° West LA on Figure 24) it is about 2.00 pm. Kick-off is often at about an hour later; the game lasts at least three hours and these arrangements enable most of it to be 'prime time' on US television.

(b) If the kick-off is carried live by British TV, what time is it broadcast i.e. British time?

SAQ 13 Look at Figure 25. A sailor decides to sail south in the Atlantic Ocean, along a line of longitude 20° W, from Iceland 65° N, to the Cape Verde Islands 15° N.

FIGURE 25 Sailing along longitude 20° W in the Atlantic from Iceland to Cape Verde.

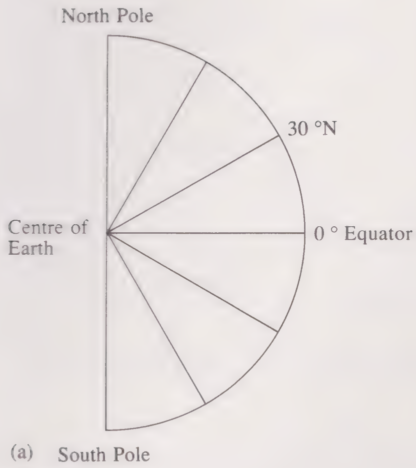
(a) What proportion of the Earth's circumference is the journey?

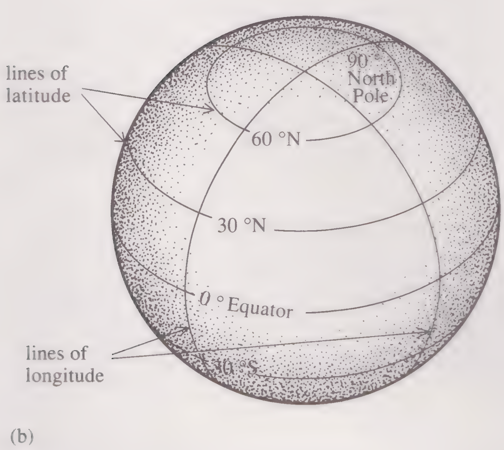
(b) What is the distance sailed? (Assume that the radius of the Earth is 6 370 km.)

(c) Does the sailor have to adjust his watch?

2.4 LABELLING LINES OF LATITUDE

The line of reference for latitude is the Equator, 0° and lines of latitude are marked north and south of it, shown in Figure 26. Degrees of latitude refer to the angle between a line through the Equator and another radius, on Figure 26a one is marked at 30° North.





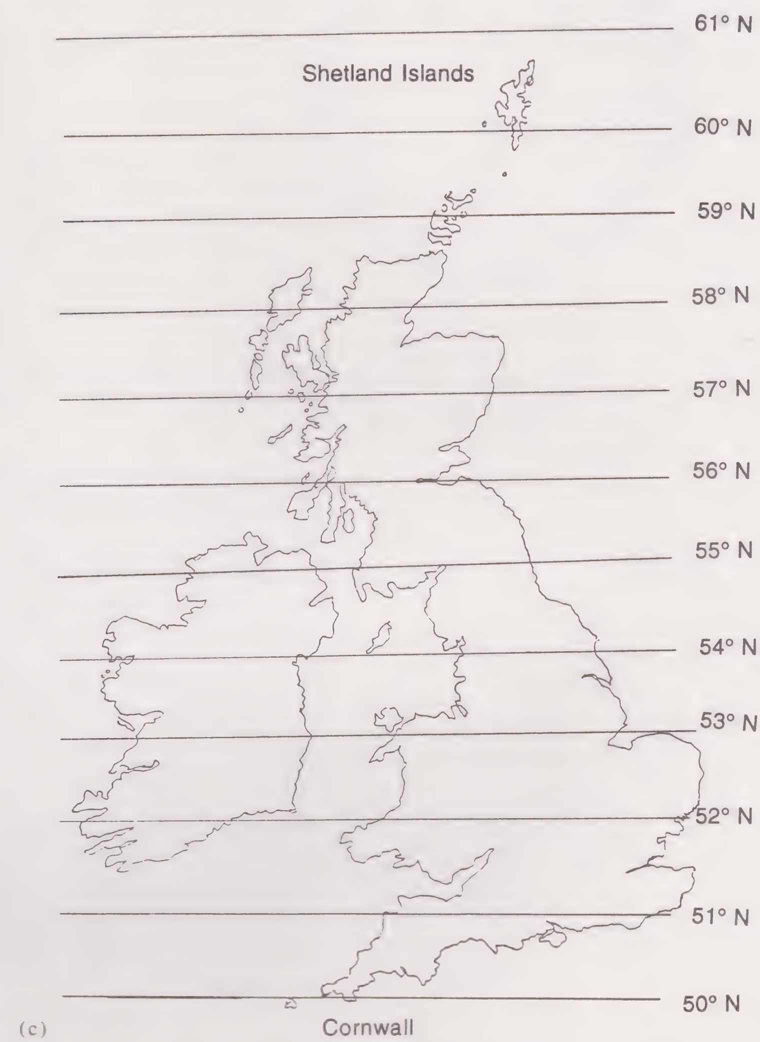


FIGURE 26 Labelling lines of latitude (a) Half the Earth, cut vertically, showing the Equator and radius to 30° N (b) Perspective view (c) Location of Britain.

Now look at Figure 26c.

□ Between which lines of latitude does Britain lie?

■ Britain lies between 50° N (Cornwall) and 61° N (Shetland Islands), shown in Figure 26c.

□ What is the highest value that a line of latitude can have, and where does this happen?

■ 90° North or 90° South, and these points are the North Pole (see Figure 26b) and the South Pole.

Now we have a network of lines about the Earth, of latitude and longitude, and these can be used to locate places. However, trying to represent the three-dimensional curved surfaces of the Earth on flat, two dimensional paper is difficult. In order to have a 'flat map' that can show the whole area of the Earth, rather than the partial areas in previous figures, we are going to flatten out the surface of the spherical Earth. If you have another orange you can try the procedure for yourself, but otherwise, just follow how it is done.

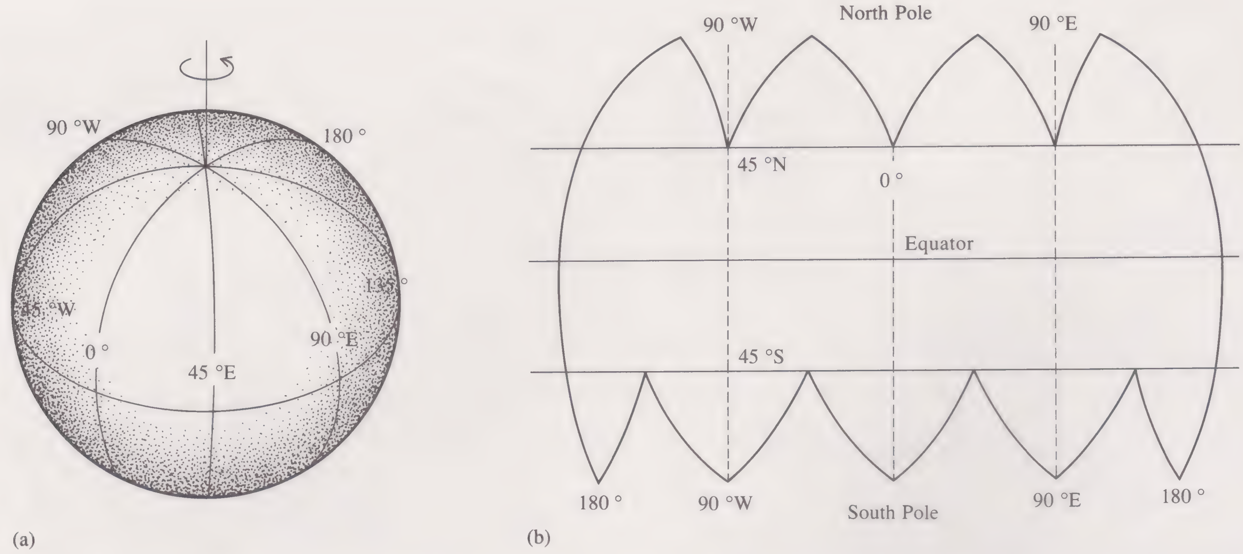


FIGURE 27 (a) Earth with 45° longitude marked, (b) Earth cut along lines of !ongitude & opened out.

Mark some lines of latitude and longitude on the peel, say at 45° intervals, that is, like Figure 27a. Cut through the peel in the following way, in order to open the skin as in Figure 27b. Cut from the North Pole to the South Pole along longitude 180°. Then, cut from the North Pole to 45° N, along longitudes 90° W, 0° and 90° E. Next, cut from the South Pole to 45° S, along 45° E, 45° W, 135° E and 135° W. You should now be able to unpeel the skin and flatten it.

If you did flatten an orange peel, you will realise the inherent problems of representing the curved surface of the Earth on a flat sheet of paper; there are bulges in the peel that have to be 'shrunk' into the flat area close to the Equator and there are 'gaps' towards the poles.

Although world maps that look like Figure 27b can be found in atlases, they are often like Figure 28, where the areas near the poles have been 'stretched' to fill a rectangular space. As a result, countries near the poles look much bigger, in comparison with those near the Equator, than they really are. Reforming a rectangular map into a volume would result in an Earth the shape of a can, not an orange!

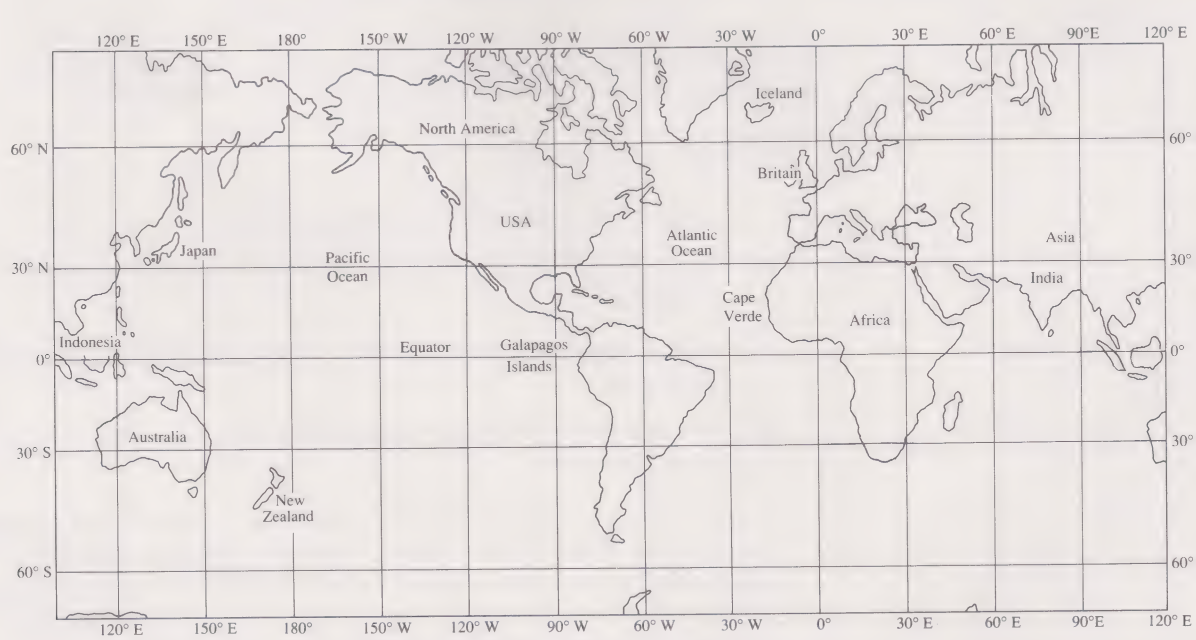


FIGURE 28 World map for use with SAQ 14.

You will remember that when we plotted points on a graph in Module 7, the x-coordinate (value) was given before the y-coordinate. In contrast, when giving the latitude and longitude of places the north-south value is given first, then the east-west, so that London is 52° N, 0°, and Singapore, virtually on the Equator, is 0°, 100° E.

SAQ 14 On the map in Figure 28 plot the following places:

New Orleans in the USA, 30° N, 90° W,

Cape Town in South Africa, 35° S, 20° E.

2.5 TRIANGLES

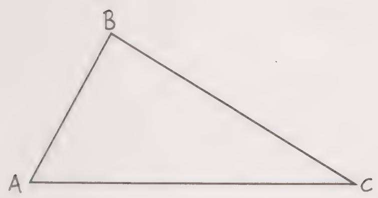


FIGURE 29 A triangle.

A triangle is a figure bounded by three straight lines which are called the *sides* of the triangle (Figure 29). These sides form three angles (hence the name of the figure) and the vertices of a triangle are usually labelled with capital letters. You first met a triangle in Module 3, when you calculated the length of roof rafters.

The triangle in Figure 29 would be called 'the triangle ABC' (sometimes written ΔABC) and the angle at the vertex A is sometimes written as $\angle$BAC (or $\angle$CAB). Notice that none of the angles in ΔABC is a right angle, they are all less than 90°. Angles smaller than right angles are called **acute** angles.

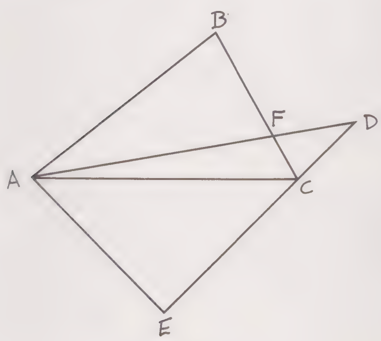


FIGURE 30 Triangles.

SAQ 15 How many triangles are there in Figure 30? Identify them, using the notation introduced above.

SAQ 16 How many acute angles are there with vertex A? Identify them, using the notation of Section 2.5.

2.6 THE ANGLE-SUM PROPERTY OF A TRIANGLE

To carry out the experiment involved in this Section, you will need some scrap paper and a pair of scissors.

Draw a triangle, similar in shape to that shown in Figure 31a, on a piece of rough paper and mark each vertex with a different symbol. Cut along the sides of the triangle. Tear off the three vertices and lay them side by side and touching one another (in any order), as shown in Figures 31b or 31c.

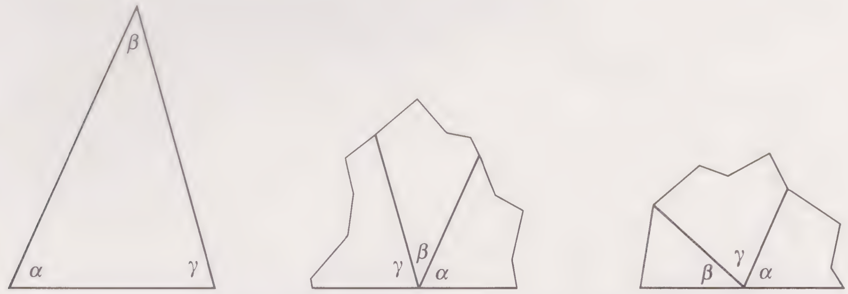


FIGURE 31 (a) Triangle with angles α, β, γ, marked. (b) and (c) Angles arranged on a straight line.

□ What do you notice about the sum of these angles?

■ You should find that, when added up, these vertices form a single straight line, similar to Figure 16b, demonstrating that the sum of the angles of your triangle is 180°.

Repeat this experiment a number of times, using triangles of different shapes: in particular you should try a triangle containing a right angle and one containing an **obtuse** angle (larger than 90°). In all cases you will find that the angles add up to 180°.

This is an important mathematical rule and you should try to remember it.

> The angles of a triangle always add up to 180°

Given two angles of a triangle, you should now be able to use this *angle-sum property* to calculate the magnitude of the third angle.

SAQ 17 Two angles of a triangle measure 25° and 65°. What is the size of the third angle? You can draw this triangle using a protractor to check your answer.

3 NAVIGATING AROUND—AT LAST

You may be wondering what all this has to do with navigation, but lines of latitude and longitude, the angle of the sun in the sky and the radius of the Earth all play important roles.

All lines of longitude and the Equator are whole circumferences of the Earth (these are sometimes called 'Great Circles'). Great Circles, complete circumferences, can also be drawn at angles to the lines of latitude and longitude, as shown on Figure 32. These circles are extremely important to airlines, when planning long routes around the Earth as the Great Circle (or an arc of one) is often the shortest distance. This is one reason why flying the 'polar route' to Japan from London is shorter.

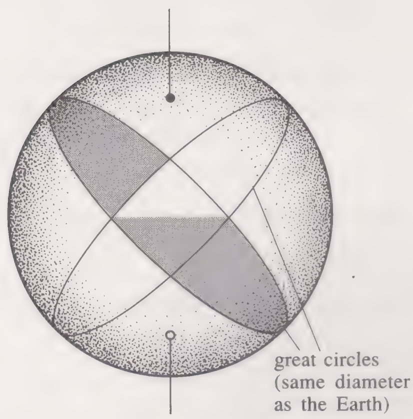


FIGURE 32 Great Circles.

But we will use simpler examples. Imagine that you are a keen sailor and have persuaded friends or family to accompany you on a voyage. You have decided that you will sail along the Equator from the Galapagos Islands off the coast of South America 0° N 90° W until you get to Indonesia, 0° N 120° E. This is shown in Figure 33.

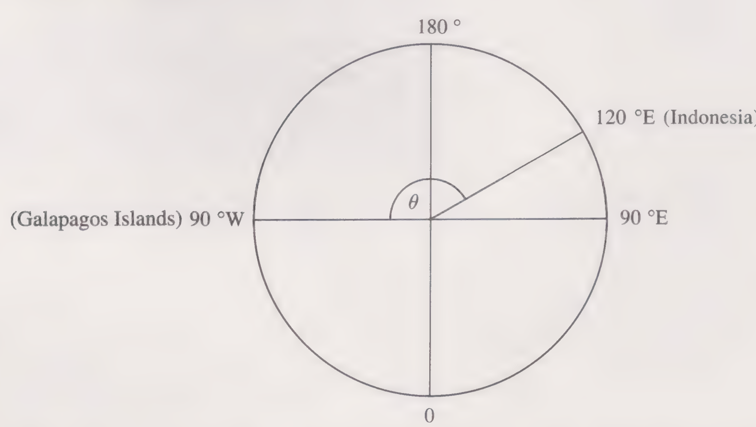


FIGURE 33 View of the Equator from the North Pole, to show the distance between Galapagos and Indonesia.

□ Draw a circle to represent the Earth and draw the angle between the two places. If the radius of the Earth is 6 370 km, how far will you have to sail?

■ Figure 33 shows the angle, 150°. The distance to be sailed is represented by:

$$\frac{150^\circ}{360^\circ} = \frac{\text{distance}}{\text{circumference}}$$

$$\frac{15}{36} = \frac{\text{distance}}{2 \times \pi \times 6\,370}$$

Rearranging this so that distance is the subject of the equation:

$$\text{distance} = \frac{15 \times 2 \times \pi \times 6\,370}{36} = 16\,676.6\,\text{km or } 16\,700\,\text{km (to 3 sig figs)}$$

This is nearly half way around the circumference of the Earth (40 000 km).

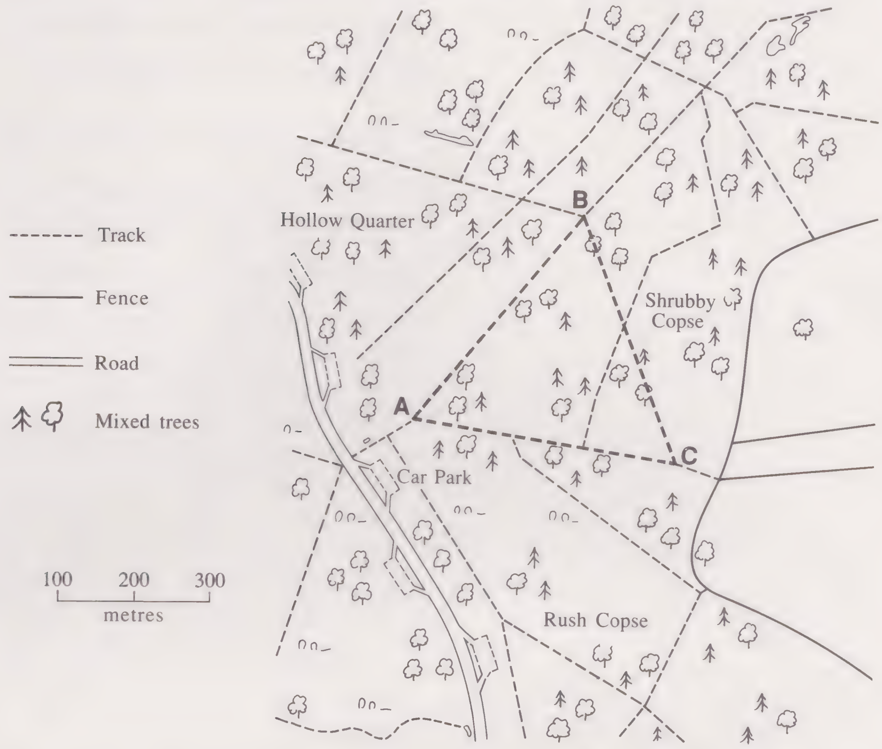


FIGURE 34 Map of a wood, part of an orienteering course.

Perhaps a closer-to-home example is more appropriate. You decide to take a walk in a wood near your home. You have a map (Figure 34) and a compass (a protractor with a needle that points to the north) that means you can measure angles. You start to walk north-eastwards from the car park at A, along the marked track. The map shows that there are plenty of tracks through the wood and you plan to walk from A to B to C, in a triangle. You cannot get lost, as by counting the turnings left and right, you should always know where you are—simple! After 150 metres a broad track crosses yours. This is not marked on the map, which you now examine and find was last surveyed in 1964. You begin to realise that some new tracks have been made, to take out timber from the wood and perhaps others have ceased to exist by having become overgrown. What do you do? One solution is to turn around and return immediately to the car park. Instead you use your knowledge of distances and angles, to work out where you turn and by how much.

☐ How far is it from A to B? (Use the scale marked on the map.)

■ It is about 350 metres.

So, after about 350 metres you need to turn right, at the point B, in order to get to C. By how much do you need to turn, or, stated another way by what angle? You need to turn through about 120° to face towards C.

☐ What is the angle ABC?

■ It is about 60°.

☐ How far is it from B to C?

■ It is about 350 metres.

Again, you need to turn through 120° to face A and after another 350 metres should be back at the car park. But if you were to take a wrong turning, or you turn after the wrong distance, it could take some time to establish where you are, and how to extract yourself. With few landmarks or open views a pleasant Sunday walk could become a nightmare.

The triangle ABC is a special type:

□ Can you identify two special features about triangle ABC?

■ The three angles are about 60° and the sides are the same length (in this case 350 m, but they could be any length).

This type of triangle is called **equilateral**, meaning equal sides and equal angles.

Taking another walk, you set out from a straight E–W road, to climb a hill. You go up a track at an angle of 60° to the road, shown in Figure 35. After 1 km your companion twists an ankle and you decide to go for help. Draw the road and AB shown in Figure 35 on a piece of graph paper allowing 10 cm for 1km along AB and measuring the angle BAC with a protractor.

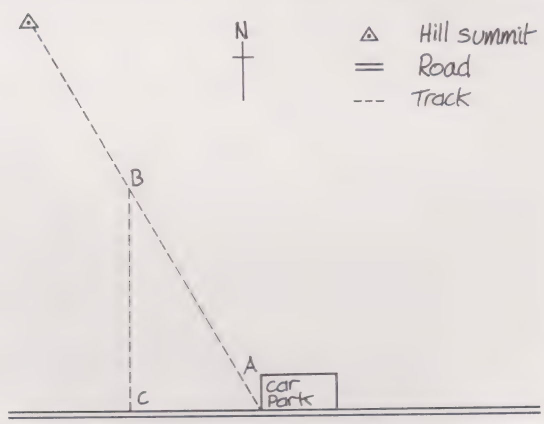


FIGURE 35 A walk up a hill from a road.

□ In what direction is the most direct route to the road? (Find the shortest distance between B and C on your diagram.)

■ The most direct route to the road is directly south, along the dotted line BC. This route will meet the road at a right angle, at ACB. Confirm this by measuring ACB with a protractor.

□ If the angle BAC is 60°, what is angle ABC ?

■ Using the angle sum of a triangle it must be $180 - (90 + 60) = 30$ degrees

□ How far along the road is C from A when you come down the hill? If you have drawn the triangle on graph paper, measure off the distance.

■ It is 0.5 km (500 m)

You can confirm this by folding a piece of paper and cutting out a triangle with angles 90°, 60° and 30°, as shown in Figure 36a. The hypotenuse should be, say, 10 cm and this represents one kilometre. Open out the piece of paper that you have cut out.

□ What shape is the cut-out?

■ It is a triangle, shown in Figure 36b.

Measure the lengths of the sides and the angles.

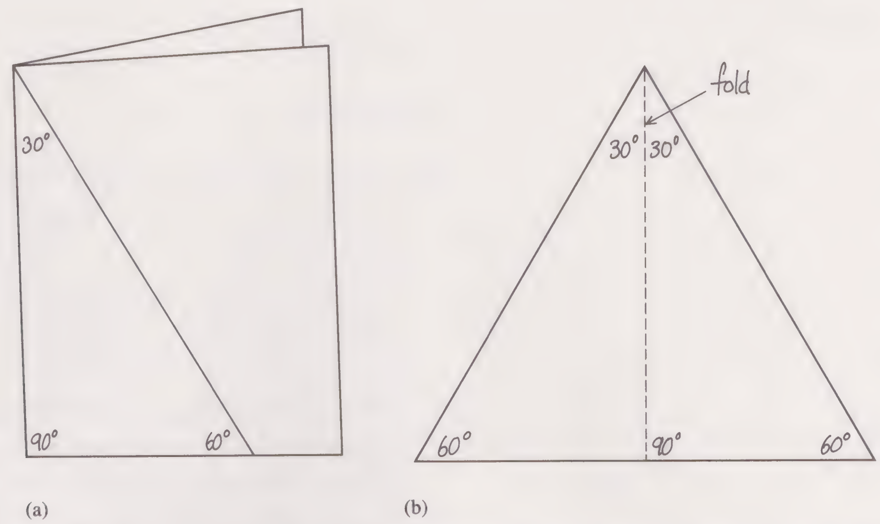


FIGURE 36 (a) A triangle marked on a folded piece of paper (b) The triangle in Figure 36a, opened out.

□ What type of triangle is it?

■ It is an equilateral triangle, with the angles each 60° and the sides all the same length. This explains why AC in Figure 35 is 0.5 km, half the length of the side of an equilateral triangle with sides of 1 km.

This demonstration shows that you can divide an equilateral triangle into two equal right-angled triangles, or put another way, if you divide an angle of an equilateral triangle in half, the line will be at right angles to the other side (see Figure 36b).

Once you have right angled triangles, you can do all sorts of interesting calculations to help your navigation; that's the subject of the next Module.

4 OVERVIEW

SUMMARY

These are the concepts you have learnt about in this Module:

- The Earth is approximately spherical with a spherical internal structure of mantle and core.
- Lines of latitude and longitude can be used to create a map of the world.
- Angles can be measured in degrees.
- Bearings can be used to plot direction.
- Triangles have three sides and three angles, the sum of the angles is 180°.
- Equilateral triangles have three equal sides and three equal angles, and can be divided into two equal right-angled triangles.
- The ratio of the circumference of a circle to its diameter is π (called 'pi').

SKILLS

Now that you have completed this Module you should be able to:

- measure angles using a protractor
- draw circles with a pair of compasses
- calculate the circumference or area of a circle (or the radius, given the area), using the formulae $C = 2\pi r$ and $A = \pi r^2$
- calculate the volume and surface area of a sphere using the formulae:

  $$V = \frac{4}{3}\pi r^3 \text{ and } A = 4\pi r^2$$

- calculate distances along portions of the Earth's circumference (arcs)
- divide by fractions.

APPENDIX 1: EXPLANATION OF TERMS USED

ACUTE (angle) An angle measuring less than a right angle (90°).

ANGLE The measurement of an amount of rotation.

ARC (of a circle) Portion of the circumference.

AXIS An imaginary line about which an object rotates.

CIRCUMFERENCE A line at a fixed distance from a point (the centre of a circle) or, the 'enclosing line' of a circle.

COMPASSES (pair) An instrument used to draw circles.

COMPASS An instrument with a needle that indicates the direction of North and is graduated in degrees, so can be used as a protractor.

CONSTANT (value) A relationship that is the same for all situations, for example the ratio of the diameter to the circumference of a circle (π).

CORE (of the Earth) The central volume of the Earth, believed to have a liquid outer part and a solid inner part.

CRUST The outer skin of the solid earth that varies in thickness, being thinner under the ocean and thicker under the continents.

DIAMETER A line across a circle, from circumference to circumference, passing through the centre. Twice the radius of the circle.

EQUATOR The line of latitude midway between the North and South Poles, given as 0°.

EQUILATERAL TRIANGLE A triangle with equal sides and equal angles (= 60°).

GREENWICH MERIDIAN Line of longitude through Greenwich which acts as a baseline. Other lines of longitude are measured E or W of this line, which is given as 0°.

INTERNATIONAL DATE LINE Line of longitude 180°. Crossing this line from W to E you move to the next day; crossing E to W you move to the previous day.

LATITUDE (lines of) Circles around the Earth that are parallel to the Equator 0°. They are measured in degrees North or South of the Equator.

LINEAR Values or measurements of a line, or in one dimension e.g. m, cm, mm.

LONGITUDE (lines of) Circles around the Earth that pass through the North and South Poles. All are circumferences of the whole Earth.

MANTLE The thick layer of solid rock inside the Earth immediately below the crust.

MAP The representation on a flat surface (e.g. a piece of paper), of part of the Earth's surface. May be a small area, e.g. a town, or a large area, e.g. a continent or the whole Earth.

MODEL A representation of an object.

OBTUSE (angle) An angle greater than a right angle.

π (pi) The ratio of the circumference of a circle to its diameter. The value is close to 22/7, or, to 6 decimal places is 3.141592.

PARALLEL A line at a constant distance from another.

PERSPECTIVE (view) A view giving the relative positions and sizes of objects, as they appear from a particular point.

PROTRACTOR An instrument graduated in degrees for measuring the size of angles.

RADIUS A line joining the centre of a circle to the circumference.

SPHERE An object that is circular in all directions, such as a tennis ball.

SUBSTITUTING Putting in known values.

VERTEX The common point where two lines meet, forming an angle.

SAQ ANSWERS AND COMMENTS

SAQ 1

(a) Thickness of ocean crust = 5 km
Earth's radius = 6 370 km

$$\text{Ratio} = \frac{5\text{ km}}{6\,370\text{ km}} = \frac{1}{1\,274}$$

$$\text{As a percentage} = \frac{1}{1\,274} \times 100\% = 0.078\%$$

(b) Thickness of continental crust = 35 km
Earth's radius = 6 370 km

$$\text{Ratio} = \frac{35\text{ km}}{6\,370\text{ km}}$$

$$\text{As a percentage} = \frac{35 \times 100}{6\,370}\% = 0.549\%$$

So, even on continents, the rocky sphere is about half of one percent of the Earth's radius — quite thin!

SAQ 2

$$\text{Ratio} = \frac{\text{thickness of atmosphere}}{\text{Earth's radius}}$$

$$= \frac{8\cancel{0}\text{ km}}{6\,37\cancel{0}\text{ km}}$$

Divide top and bottom by 10, then divide as in Box 7

$$= \frac{1}{79.625}$$

$$\approx \frac{1}{80}$$

$$\approx 1{:}80$$

BOX 7		
Press	637	
press	÷	
press	8	
press	=	79.625 should appear

SAQ 3

To find the circumference of the tin:

$$C = 2\pi r$$

$$= (2 \times \frac{22}{7} \times 10)\text{ cm}$$

$= 62.9$ cm (to three figures)

$= 63$ cm, to the nearest cm.

If you used your calculator, and used the key π, you will have a slightly different answer.

SAQ 4

Distance travelled = 25 737 km = circumference of circle

$$C = 2\pi r$$

Rearranging this so that r is the subject of the equation:

$$r = \frac{C}{2\pi}$$

$$= \frac{25\,737}{2} \times \frac{7}{22}\text{ km}$$

$= 4\,094.5$ km (to 5 sig figs)

The radius of the circle = 4 094.5 km (to 5 sig figs).

SAQ 5

Circumference of Earth = $2\pi r$

$$= 2(\pi)6\,370\text{ km}$$

In 24 hours a place will move: $2(\pi)6\,370$ km

$$\text{in 1 hour} = \frac{2(\pi)6\,370}{24}\text{ km}$$

A place on the equator moves at 1.67×10^3 km per hour (to 3 sig figs). The keys to press are in Box 8 below.

Note: on some calculators the π button has 'exp' on the key and π beneath the key

BOX 8		
Press	2	
press	×	
press	π	(3.141 592 7 should appear)
press	×	(6.283 153 should appear)
press	6370	
press	÷	(40 023.89 should appear)
press	24	
press	=	(1 667.662 1 should appear)

$1\,667.662\,1 = 1.67 \times 10^3$ in scientific notation to 3 significant figures.

SAQ 6

(a) Area = πr^2

Radius = 21.0 km

$$\text{Area} = \frac{22}{7} \times (21)^2\text{ km}^2$$

$$= \left(\frac{22}{\cancel{7}} \times \frac{\overset{3}{\cancel{21}}}{1} \times \frac{21}{1}\right)\text{ km}^2$$

$= 22 \times 3 \times 21\text{ km}^2 = 1\,386\text{ km}^2$

$= 1\,390\text{ km}^2$ (to 3 sig figures).

(b) Area = πr^2

$= 1\,385.442\,4\text{ km}^2$

$= 1\,390\text{ km}^2$ (to 3 sig figs)

SAQ 7

(a) $A = \pi r^2$

$401\text{ cm}^2 = \pi r^2$

$r^2 = \frac{401}{\pi}\text{cm}^2$ rearranging, Eqn (1)

$$r^2 = \frac{401}{1} \times \frac{7}{22}$$

$r^2 = 127.590\,909\,1\text{ cm}^2$

Press the square root button, so

$$r = \sqrt{127.590\,909\,1\text{ cm}^2} = 11.295\,614\,6\text{ cm}$$

Diameter is twice the radius, so

Diameter = (11.295 614 6 × 2) cm

= 22.591 229 19 cm

Because the area (401 cm^2) is given using only three significant figures, the diameter is also expressed to three significant figures. So diameter = 22.6 cm.

(b) From Eqn (1), $r^2 = \frac{401}{\pi}$

so, r^2 = 127.642 26 cm^2

Press √ key

r = 11.297 888 cm

Diameter is twice the radius, so,

Diameter = 22.595 775 cm

= 22.6 cm (to three sig figs)

The calculator keys to press are in Box 9.

BOX 9		
Press	401	
press	÷	
press	π	
(or 'exp' button)		
press	=	(127.642 26 should appear)
press	√	(11.297 888 should appear)
press	×	
press	2	
press	=	(22.595 775 should appear)

SAQ 8 Square area: perimeter = 400 m

each side is = 100 m, i.e. $\frac{400}{4}$ m

area of square (100 × 100) m^2

= 10 000 m^2

Round area: circumference

= 400 m

$2\pi r$ = 400 m

$r = \frac{400}{2\pi}$ m

area = πr^2

$= \pi \times \left(\frac{400}{2\pi}\right)^2 m^2$

$= \pi \times \frac{400}{2\pi} \times \frac{400}{2\pi} m^2$

Note when handling a complicated fraction, such as the one above, reduce it to its lowest terms by cancelling (see page 7), before using your calculator. That way you will have less key strokes to make and you are less likely to make errors.

In this case the equation can be simplified to:

$$\text{area} = \cancel{\pi} \times \frac{\overset{200}{\cancel{400}}}{\cancel{2}\cancel{\pi}} \times \frac{\overset{200}{\cancel{400}}}{\cancel{2}\pi} m^2 = \frac{40\,000}{\pi} m^2$$

= 12 732.395 m^2

= 12 732 m^2 (to 5 figures)

The calculator keys to press are in Box 10.

So the round area is bigger than the square area by

12 732 m^2 − 10 000 m^2 = 2 730 m^2 (to 3 sig figs),

and the sheep would have more space and more grass in the circular area.

BOX 10		
Press	40 000	
press	÷	
press	π	
press	=	(12 732.395 should appear)

SAQ 9 Volume of Earth = 1.08×10^{12} km^3

or $\frac{4}{3}\pi (258.47 \times 10^9)$ km^3

(a) Radius of core = (6 370 − 2 900) km = 3 470 km

Volume of core = $\frac{4}{3}\pi (3.47 \times 10^3)^3$ $(km)^3$

$= \frac{4}{3}\pi (41.78 \times 10^9)$ km^3

Ratio of core to whole Earth

$$= \frac{\frac{\cancel{4}}{\cancel{3}}\cancel{\pi}(41.78 \times \cancel{10^9})\, \cancel{km^3}}{\frac{\cancel{4}}{\cancel{3}}\cancel{\pi}(258.47 \times \cancel{10^9})\, \cancel{km^3}}$$

Note when the units at the top and bottom of a fraction are the same these cancel out.

$$= \frac{41.78}{258.47}$$

As a percentage

$$= \frac{41.78 \times 100}{258.47}\% = 16.2\% \text{ to 1 dec place (3 sig figs)}$$

(b) Volume of mantle

= volume of Earth − volume of core

$= \frac{4}{3}\pi (258.47 \times 10^9)\, km^3 - \frac{4}{3}\pi (41.78 \times 10^9)\, km^3$

$= \frac{4}{3}\pi (258.47 - 41.78)\, 10^9\, km^3$

$= \frac{4}{3}\pi (216.69 \times 10^9)\, km^3$

Volume of mantle to volume of Earth

$$= \frac{\frac{\cancel{4}}{\cancel{3}}\cancel{\pi}(216.69 \times \cancel{10^9})\, \cancel{km^3}}{\frac{\cancel{4}}{\cancel{3}}\cancel{\pi}(258.47 \times \cancel{10^9})\, \cancel{km^3}}$$

As a percentage

$$= \frac{216.69 \times 100}{258.47}\% = 83.8\% \text{ to 1 dec place (3 sig figs)}$$

SAQ 10

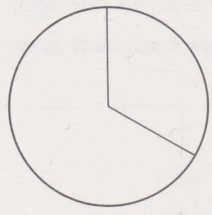
p = r; q = s; t = v; u = w;

Also

p = r = t = v and q = s = u = w

SAQ 11

(a) Figure of clock to show angle between hands.



(b) By measurement, or that this angle is 1/3 circle

Angle = 120° or $\left(\frac{360}{3}\right)$

(c) As a bearing this is 120°

SAQ 12

(a) New Orleans' longitude is different by 90°.
At 1 hr for 15° of longitude, time difference

$= \frac{90}{15}$ hrs

= 6 hrs.

(b) Super Bowl TV transmission will be 6 hrs different in London. As London is further East, the time will be later i.e. 16.00 + 6 hrs = 22.00 or 10.00pm.
Kick off is one hour later, i.e. 11.00pm British time.

Note from the map NY has 'later' time than NO, so London must be 'later'.

SAQ 13

(a) Lines of latitude through 20° W
Angle between Iceland & Cape Verde = 50°

Recall that the Earth's circumference is $2\pi r$ and we are measuring the proportion of 50°/360° of the total.

$$\frac{50}{360} = \frac{5}{36}$$

(b) Distance travelled:

$$\frac{\text{Distance travelled}}{\text{Circumference}} = \frac{5}{36}$$

$$\frac{\text{Distance}}{2 \times \pi \times 6370} = \frac{5}{36}$$

Rearranging:

$$36 \times \text{Distance} = 5 \times 2 \times \pi \times 6\,370 \text{ km}$$

$$\text{Distance} = \frac{5 \times 2 \times \pi \times 6\,370}{36} \text{km}$$

= 5 558.873 7 km

= 5 560 km (3 sig figs).

If you did not get this answer, check your method with that given in Box 11.

BOX 11

Press	10	
press	×	
press	π	
press	×	
press	6 370	
press	÷	(200 119.45 will appear)
press	36	
press	=	(5 558.873 7 will appear)

(c) The sailor does not need to adjust his watch, as this is only required when travelling East–West (or West–East), not in the North–South direction. Everywhere on the same line of longitude has the same 'time'.

SAQ 14 See map Figure 37.

SAQ 15 Seven
ABC, ADE, ABF, AFC, FDC, ACE, ADC.

SAQ 16 Six:
BAF (or BAD), FAC (or DAC), CAE, BAC, FAE (or DAE), BAE.

SAQ 17 Angle = 180 − (65 + 25) = 90°

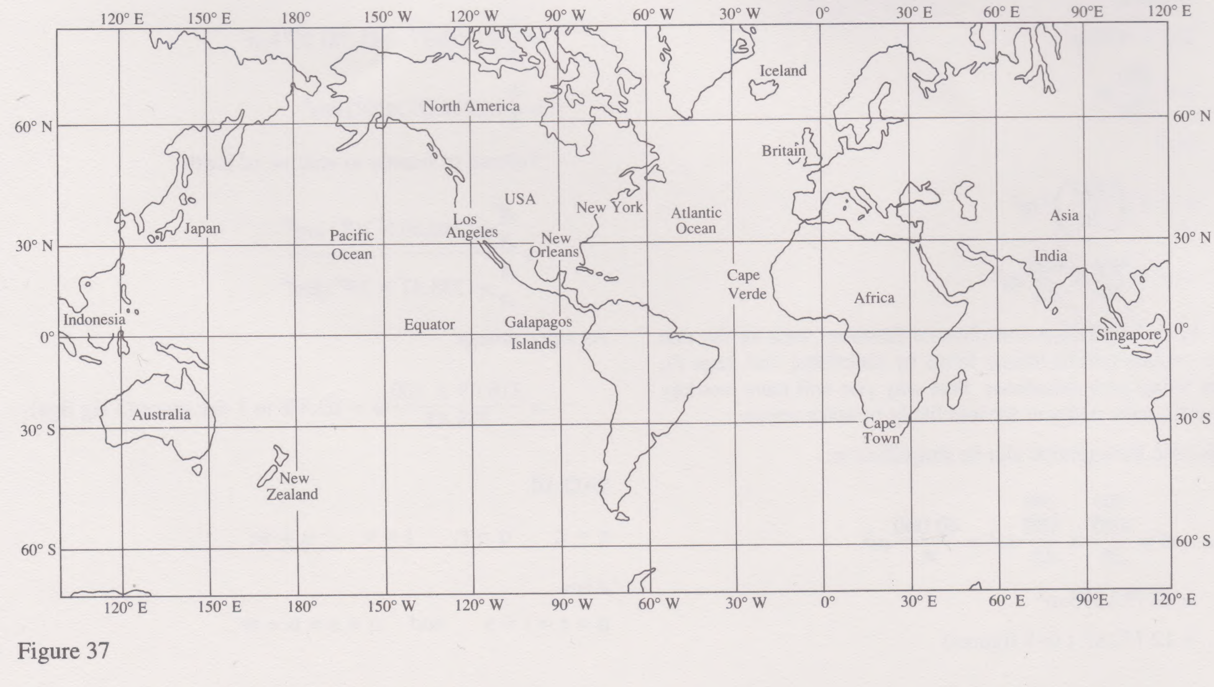


Figure 37